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The scribe could perform quickly the operations by using extensive tables. For instance the Egyptian Mathematical Leather Roll (c.a. 1600 BC) contains a short version of addition tables.

Egyptians could also solve linear equations.

"Problem 19 of the Moscow Papyrus" asks to find the number such that if it is taken $1\frac{1}{2}$ times and then 4 is added, then the sum is 10.

The scribe procedure is: "Calculate the excess of this 10 over 4. The result is 6. You operate on $1\frac{1}{2}$ to find 1. The result is $2/3$. You take $2/3$ of this 6. The result is 4. Behold, 4 says it. You will find that this is correct."

In our math modern notation he had to solve the equation $1\frac{1}{2}x + 4 = 10$.

Similarly "Problem 35" of the Rhind Mathematical Papyrus asks to find the size of a scoop that fills a 1-hekat measure in $3\frac{1}{3}$ trips.

The scribe solve the equation [with our

notation it would be written as $3\frac{1}{3}x = 1$]
by dividing 1 by $3\frac{1}{3}$. He writes the answer as $\overline{5}\ \overline{10}$ and then proceeds to prove that the result is correct.

Another common way to solve linear equations is by <u>false position</u>.

"<u>Problem 26</u> of <u>the Rhind Mathematical Papyrus</u>" asks for a quantity that when it is added to $\frac{1}{4}$ of itself, the result is 15.

(With our notation we need to solve $x + \frac{1}{4}x = 15$.)

The scribe would do as follows: "Assume the answer is 4, Then $1\overline{4}$ of 4 is 5. ... Multiply 5 so as to get 15. The answer is 3. Multiply 3 by 4. The answer is 12."

We observe that (perhaps) the first guess is 4 because $\frac{1}{4}\cdot 4 = 1$ is an integer. Then the scribe notes that $4 + \frac{1}{4}\cdot 4 = 5$ and then checks that the ratio $15/5 = 3$ and multiply 4 by 3 so to get 12.

Thus is clear that Egyptians understood the basic ideas of proportionality. There is never a proof ... everything works out!

The method of false position is also used in the only existing problem involving a quadratic equation (in the Berlin Papyrus)

This papyrus has a problem in which a square are of 100 square cubits is to be divided into 2 other squares, where the ratio of the sides of the 2 needed squares is 1 to 3/4.

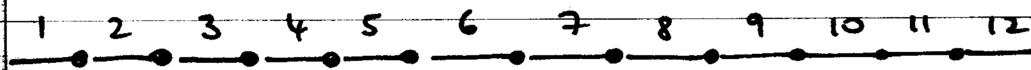
The scribes actually begins by assuming that in fact the sides are 1 and $\frac{3}{4}$. Then he calculates the sum of the areas of these 2 squares: $1^2 + \left(\frac{3}{4}\right)^2 = 1\frac{9}{16}$.
The desired area is though 100.
He knows that he cannot compare the areas directly ... but must compare the sides. So he computes the square roots:

$$100 \rightsquigarrow 10$$

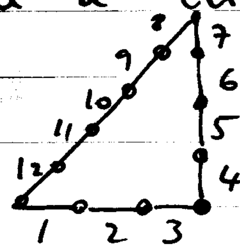
$$1\frac{9}{16} \rightsquigarrow 1\frac{1}{4}$$

Since 10 is 8 times as large as $1\frac{1}{4}$, then the scribes concludes that the sides must be $8 (= 8 \times 1)$ and $6 \left(= 8 \times \frac{3}{4}\right)$ cubits respectively.

In the previous problem we stumbled on Pythagoras Theorem : $10^2 = 8^2 + 6^2$.
Now, <u>tradition</u> says that Egyptian architects used a clever device for making a right triangle. They would tie 11 equally spaced knots on a rope



and then form a triangle with sides (3, 4, 5)



Implicit in this construction is the fact that $3^2 + 4^2 = 5^2$. It is doubtful that they possessed a broader understanding. They never gave an indication of how they might prove this relationship. Perhaps they just stumbled on it.

However, Egyptian scribes knew how to calculate the areas of rectangles, triangles and trapezoids.

"<u>Problem 50 of the Rhind Mathematical Papyrus</u>" reads as follows: " Example of a round field of diameter 9. What is its area? Take away $\frac{1}{9}$ of the diameter the remainder is 8. Multiply 8 times

8; it makes 64. Therefore, the area is 64."

In other words $\quad$ Area of circle $= \left(d - \frac{d}{9}\right)^2 = \left(\frac{8}{9}d\right)^2$

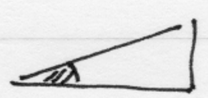
A quick comparison to our formula :

$$\text{Area of circle} = \pi r^2 \quad \left[= \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4} d^2\right]$$

gives that Egyptians used an approximation for π of $\quad \frac{256}{81} = 3.16049$

Because one of the prominent form of building in Egypt was the <u>pyramid</u> one would expect them to know a formula for the volume of a pyramid .
<u>Such a formula</u> does <u>not</u> appear in any surviving document.

There are several problems dealing with the <u>slope</u> of a pyramid. This is measured as so many horizontal units to one vertical unit rise :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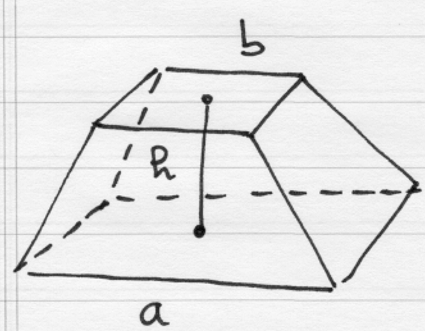
Well ... this actually corresponds to our <u>Cotangent</u> of the angle.

Anyway, the <u>Moscow Papyrus</u> has a fascinating problem containing the formula for the volume of a truncated pyramid.

"Problem 14 of the Moscow Papyrus":
If someone says to you: a truncated pyramid of 6 for the height by 4 on the base by 2 on the top, you are to square this 4, the result is 16. You are to double 4; the result is 8. You are to square this 2; the result is 4. You are to add the 16 and the 8 and the 4; the result is 28. You are to take $\frac{1}{3}$ of 6, the result is 2. You are to take 28 two times; the result is 56, Behold, the volume is 56. You will find that this is correct."



Our modern formula is

$$V = \frac{1}{3} h \left(a^2 + ab + b^2 \right)$$

Comment: certainly by setting $b=0$ you would get the well-known volume of a Pyramid: $\frac{1}{3} h \cdot$ (area base)

Even if we knew the volume of a pyramid how would we justify with our math knowledge that formula?

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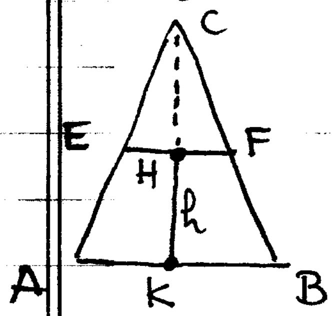
Take this section of the pyramid.



We have that $\overline{AB} = a$

$\overline{EF} = b$

whereas $\overline{CH} = ?$ $\quad \overline{CK} = \overline{CH} + h$

Thus a simple proportion gives us:

$$\frac{\overline{CH}}{\overline{EF}} = \frac{\overline{CK}}{\overline{AB}} \quad \text{or} \quad \frac{\overline{CH}}{b} = \frac{\overline{CH} + h}{a}$$

$$\text{or} \quad a\,\overline{CH} = b\,\overline{CH} + bh \quad \text{or}$$

$$\overline{CH} = \frac{b}{a-b}\,h$$

Thus, we get the volume of the truncated pyramid by subtracting the volume of the top pyramid from the big one:

$$V = \frac{1}{3}\,\overline{CK}\cdot(\overline{AB}^2) - \frac{1}{3}\,\overline{CH}\cdot(\overline{EF}^2) =$$

$$= \frac{1}{3}\left[\frac{b}{a-b}h + h\right]a^2 - \frac{1}{3}\left(\frac{b}{a-b}h\right)b^2 =$$

$$= \frac{1}{3}h\left[\left(\frac{b}{a-b}+1\right)a^2 - \frac{b^3}{a-b}\right] = \frac{1}{3}h\left[\frac{a^3}{a-b} - \frac{b^3}{a-b}\right]$$

$$= \frac{1}{3}h\left[\frac{a^3-b^3}{a-b}\right] = \frac{1}{3}h\left[\frac{\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{a-b}}\right]$$

$$= \frac{1}{3}h(a^2+ab+b^2)$$