Egyptians also succeded in "Problem 10 of the Moscow Papyrus" to find the area of the surface of a hemisphere.

As a modern formula, their result would be

$$S = 2 \cdot \left(\frac{8}{9} d\right)^2$$

or, since the area A of the circular opening of the hemisphere is given by — according to their knowledge — $A = \left(\frac{8}{9} d\right)^2$

$$S = 2 \cdot A ,$$

which is correct!

Mesopotomia

Mesopatamian civilization developed between the Tigris and Euphrates river sometimes in the 5th millenium BC.

Initially there were many small city-states. But then slowly the area was unified under a central ruler. Around 2000 BC large part of South Mesopotamia was under the Ur Dinasty (≡ Sumerian period), which produced a very centralized and burocratic state. In particular, during this period a large system of scribal schools was created in order to train members of the burocracy.

After its collapse, we went back to a city-state system... but the scribal system still remained. By 1700 BC Hammurabi, the ruler of Babylon (one of the newer city-state), had expanded to rule much of Mesopotamia and instituted a legal system to help regulate his empire.

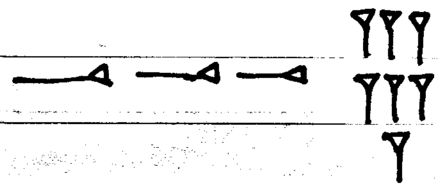
There is a third significant period in the Mesopotamian hystory, running from about 600 BC to 300AD and covering the New Babylonian Empire of Nebuchadnezzar and the following Persian and Seleucidan eras.

Of the approximatively 500,000 tablets found in the archeological sites, about 400 have been identified as strictly mathematical tablets. They can be grouped into 3 sets of tablets corresponding to those periods we mentioned above. Half of those tablets contain multiplic. tables, tables of reciprocals, tables of squares and cubes, and even tables of exponentials (probably used for problems of compounded interest

Method of Computation: The Babylonians developed a sexagesimal system. There are cuneiform symbols indicating 1, 60, 3,600 and also 60^{-1} and 60^{-2}. For instance:

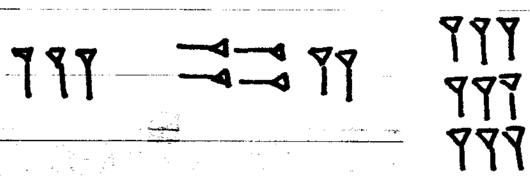
 $\equiv 1$  $\equiv 10$.

By grouping, scribe would for instance represent 37 as:

 $\equiv 37$

However their most characteristic feature was that the Sumerians indicated higher numbers by using the same symbols but indicated their value by the position. For instance they indicated the number

$$3 \times 60^2 + 42 \times 60 + 9 = 13{,}329$$



This is not different at all from our system $343 = 3 \cdot 10^2 + 4 \cdot 10 + 3$, except that we use a decimal system.

The old Babylonians did not have a symbol for zero. They often used a space if a given number was missing. There are some ambiguities though: there would not be a space at the end of a number, so it is difficult to distinguish between, say, $3 \times 60 + 42$ and $3 \times 60^2 + 42 \times 60 + 0$. The exact interpretation could be gathered by the context.

Since the Babylonian number system was a <u>place value system</u>, the actual algorithm for addition and subtraction (including carrying and borrowing) may well have been similar to our modern one.

Say:

$$\begin{array}{r} 23,\boxed{37} + \\ 41,\boxed{32} = \\ \hline \end{array} \qquad \begin{array}{l} (\equiv 23 \times 60 + 37 = 1{,}417) \\ (\equiv 41 \times 60 + 32 = 2{,}492) \end{array}$$

$$\downarrow$$

1,09 ($\equiv$ 69 in our numbering system)

The scribe would add 37 and 32 getting 69 or $1 \times 60 + 9$. Thus he would write nine and carry 1. Then he would write $23 + 41 + 1$ getting 65 or 1,05. Thus his answer would have been

$$1,05,09 \; (\equiv 1 \times 60^2 + 5 \times 60 + 9 = 3{,}909)$$

As we mentioned earlier, they had most of their tablets dedicated to extensive multiplication tables. For instance, they would have all the multiples of 9 say from 1×9 to 20×9 and

then they gave 30×9, 40×9, 50×9.
Thus to find the product 34×9 they would do $30 \times 9 + 4 \times 9$. Etc...

Fractions of course were treated as sexagesimal fractions.

Observe that not only the place value system remained a possession of mankind but also the sexagesimal system. Our present division of the hour into 60 minutes and 3600 seconds dates back to the Sumerians... as does the division of the circle in 360 degrees, each degree in 60 minutes and each minute in 60 seconds.

The choice of using 60 was perhaps due to the fact that they were trying to unify all numerical systems plus 60 has lots of divisors.

Geometry: The next set of tablets dates back to the period of King Hammurabi. By then, the Babylonians had developed procedures for determining areas and volumes of various kinds of figures. The reasoning behind many of the procedures they devised go back no longer to the original accountancy tradition, but to the <u>cut-and-paste geometry of surveyors</u> (who had to measure the fields). Not only did these manipulations of squares and rectangles developed into procedures for determining square roots and finding Pythagorean triples, but they also developed into "<u>algebra</u>".

They presented "formulas" in terms of what we would call "<u>coefficient list</u>".

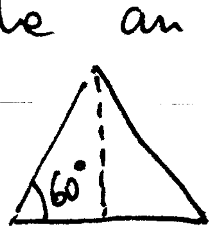
For instance, $7/8$ as the coefficient attached to the height of an equilateral triangle meant that the height of an equilateral triangle is $7/8$ of its base.

Similarly, $\frac{7}{16}$ as the coefficient attached to the area of an equilateral triangle meant that the area of an equilateral triangle is $7/16$ of the square of its side.

Note that $h = \frac{7}{8}\ell$ and $A = \frac{7}{16}\ell^2$

both include an implicit approximation.

Namely



$h = \frac{\sqrt{3}}{2}\ell \qquad A = \frac{\sqrt{3}}{2}\cdot\ell\cdot\ell\cdot\frac{1}{2} = \frac{\sqrt{3}}{4}\ell^2$

i.e. $\sqrt{3} \simeq 7/4 = 1.75$ whereas $\sqrt{3} = 1.7320$.

In each case the idea behind is that the defining component for a triangle is its side.

For a circle the Babylonians took the circumference as the defining component. Thus they gave $\frac{1}{3}$ as coefficient for the diameter and $1/12$ as the coefficient for the area of the circle.

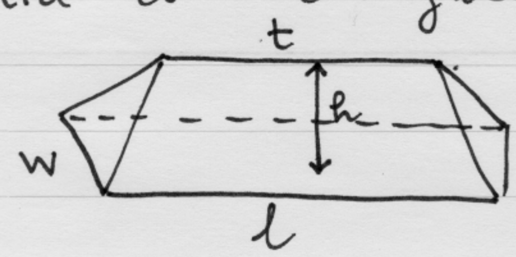
We also have a defining component for a circle: its radius. In the Babylonians formulas there is embedded another approximation:

$$d = \frac{1}{3}\,\text{circumf} \Rightarrow 2r = \frac{1}{3}(2\pi r)$$

$$A = \frac{1}{12}(\text{circumf})^2 \Rightarrow \pi r^2 = \frac{1}{12}(2\pi r)^2$$

$$\Rightarrow \pi = 3$$

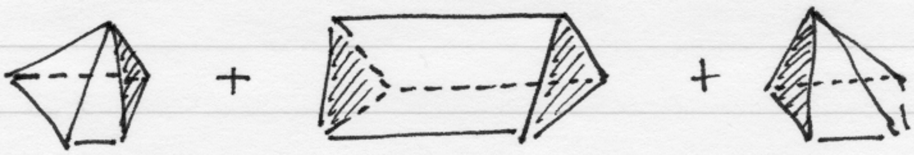
Babylonians also dealt with volumes of solids. They realized that the volume V of a block is $V = lwh$. There is no account for the volume of a pyramid, but there are several problems involving a grain pile in the shape of a rectangular pyramid with elongated apex:



Their method of solution corresponds to the formula

$$V = \frac{hw}{3}\left(l + \frac{t}{2}\right)$$

This was achieved by breaking the pile into 3 pieces:



So we perhaps might assume they knew the volume of the pyramid (set $t=0$).

There is also a tablet about the volume of a truncated pyramid with square base a^2 and top base b^2. They had

$$V = \left[\left(\frac{a+b}{2}\right)^2 + \frac{1}{3}\left(\frac{a-b}{2}\right)^2\right] h$$

which you can check that is nothing but

$$= \dots = \frac{1}{3} h \left(a^2 + ab + b^2\right)$$

as in the case of the Egyptians.

Square root and the Pythagorean Theorem:

Another algorithm that appears in their tablets is the one to compute square roots.