In solving these problems the scribe made full use of the cutting-and-paste geometry developed by the surveyors.

For example consider the problem of solving

$$x + y = b \qquad xy = c$$

(actually the real problem had $b = 6\frac{1}{2}$ and $c = 7\frac{1}{2}$

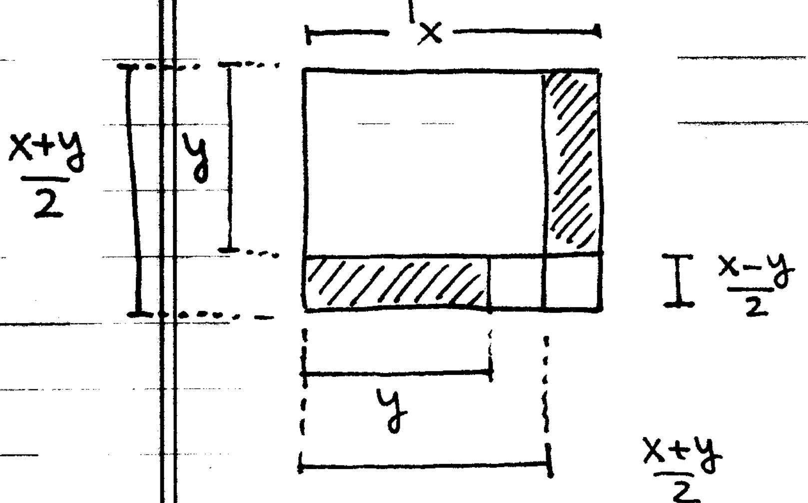
The scribe had in mind a very geometric procedure which is summarized in the identity

$$\left(\frac{x+y}{2}\right)^2 = xy + \left(\frac{x-y}{2}\right)^2 .$$

From which we get: $\frac{x+y}{2} = \frac{b}{2}$ & $\frac{x-y}{2} = \sqrt{\left(\frac{b}{2}\right)^2 - c}$

so that $x = \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}$ & $y = \frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - c}$.

Geometrically the above formula is explained in the picture:



since $x - \left(\frac{x-y}{2}\right) = \frac{x+y}{2}$

$y + \left(\frac{x-y}{2}\right) = \frac{x+y}{2}$

the square of $\frac{x+y}{2}$ exceeds xy by the square of $\frac{x-y}{2}$.

A geometric interpretation can be given to the Babylonian solution of what we would call a quadratic equation

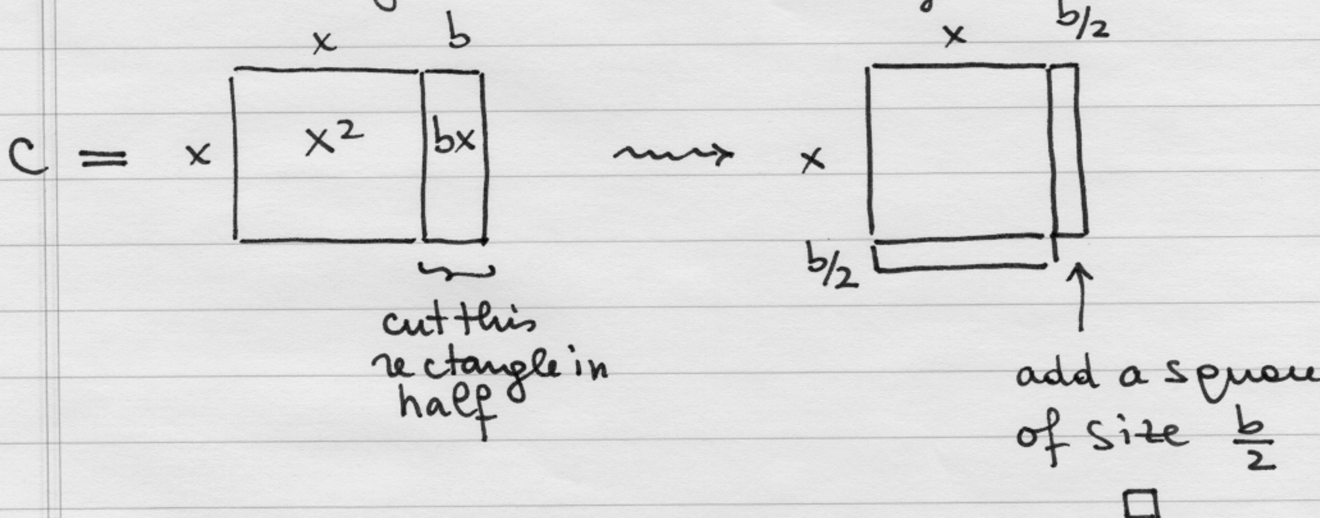
$$x^2 + bx = c.$$

Their rule of solving it translates in

$$x = \sqrt{\left(\frac{b}{2}\right)^2 + c} - \frac{b}{2}$$

Observe that the negative solution was completely ignored by the Babylonians since it made <u>no</u> geometrical sense.

Here is the geometrical meaning:



$$\therefore \quad c + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$$\therefore \quad \sqrt{c + \left(\frac{b}{2}\right)^2} = x + \frac{b}{2}$$

hence the claimed solution.

Babylonians also considered problems leading to a system of the form:

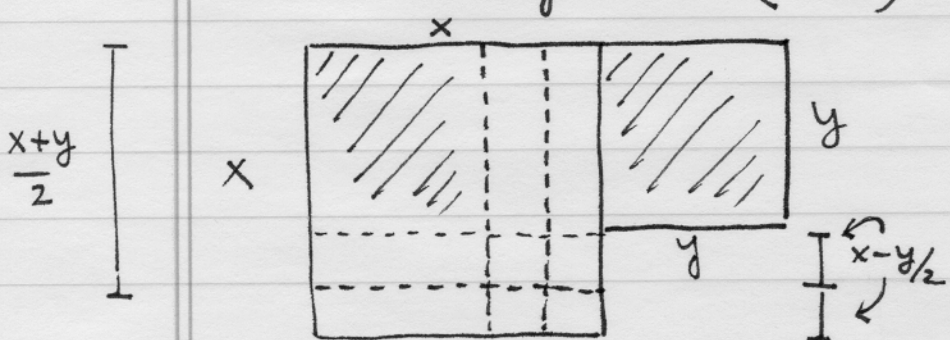
$$x^2 + y^2 = c$$
$$x - y = b$$

whose solution is describable by the modern formula

$$x = \sqrt{\frac{c}{2} - \left(\frac{b}{2}\right)^2} + \frac{b}{2} \quad \& \quad y = \sqrt{\frac{c}{2} - \left(\frac{b}{2}\right)^2} - \frac{b}{2}$$

Again one can look for a geometric interpretation. Algebraically it boils down to the formula:

$$x^2 + y^2 = 2\left(\frac{x+y}{2}\right)^2 + 2\left(\frac{x-y}{2}\right)^2 \qquad \text{or}$$



$$\text{as } x = \frac{x+y}{2} + \frac{x-y}{2}$$
$$y = \frac{x+y}{2} - \frac{x-y}{2}.$$

Also the equation $x^3 + x^2 = a$ appears in a problem which calls for the solution of:

$$xyz + xy = 1 + \frac{1}{6} \qquad y = 2/3\,x \qquad z = 12x \;.$$

This leads to $48x^3 + 4x^2 = 7$ or better to $(12x)^3 + (12x)^2 = 252$.

But Babylonians had tables for the values $a^3 + a^2$. From the table we get $12x = 6$.