Early Greek Mathematics

The last centuries of the second millenium BC witnessed many economic and political changes. Some civilizations disappeared, the power of Egypt and Babylonia waned, and new peoples (Hebrews, Assyrians, Phoenicians, and Greeks) came to the fore.

The Iron Age brought with it sweeping changes in warfare and in all pursuits that required tools. The alphabet was invented, and coins were introduced. Trade was increasingly stimulated, and geographical discoveries were made. The world was ready for a new type of civilization.

As far as Greece is concerned, it was about 1200 BC when the primitive Dorian tribes moved southward into the Greek peninsula, leaving their northern mountain for more favorable territories. Their chief tribe, the Spartans, subsequently developed the city of Sparta. Many of the former inhabitants of the invaded regions fled to Asia Minor and the Ionian islands of the Aegean Sea, where they established

Greek trading colonies. It was in these colonies, in the VI th century BC, that the Ionian school was founded, Greek philosophy blossomed, and demonstrative geometry was born.

The static outlook of the ancient Orient became impossible, and in a developing atmosphere of rationalism, men began to ask why as well as how.

Thus mathematics, in the modern sense of the word, was born in this new atmosphere of rationalism. Tradition has it that demonstrative geometry began in one of the new trading towns located on the West coast of Asia Minor. Thales of Miletus (624 – 547 BC), one of the seven wise men of antiquity, is traditionally considered the father of Greek Mathematics.

Thales seems to have spent the early part of his life as a merchant, becoming wealthy enough to devote the latter part of his life to study and some travel. It is said that Thales resided for a time in Egypt, and there he impressed the officials by determining the height of a pyramid by

comparing the length of its shadow to that of the length of the shadows of a stick of known length. Thus, he knew the basic principles of similarity.

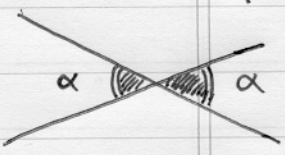
Back in Miletus, his many-sided genius won him a reputation as a statesman, counselor, engineer, businessman, philosopher, mathematician, and astronomer.

Thales is the first known individual with whom mathematical discoveries are associated. In geometry, he is credited with the following theorems:

1. A circle is bisected by any diameter

2. The base angles of an isosceles triangle are equal.

3. The vertical angles formed by two intersecting lines are equal.



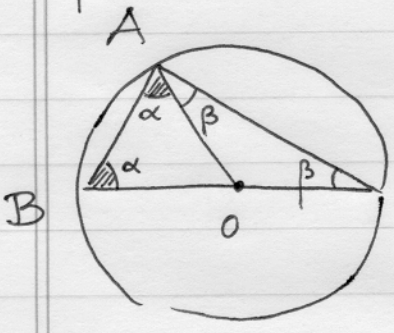
4. Two triangles are congruent if they have 2 angles and one side in each respectively equal.
(He perhaps used this result in his determination of the distance of a ship from shore.)

5. The angle sum of a triangle equals two right angles.

6. An angles inscribed in a semicircle is a right angle.

Although exactly how Thales accomplished these facts is not known, it does seem clear that he advanced some logical arguments.

For instance the following proof of 6. (see Proposition 31 in Book III of Euclid's Elements) is so simple that it is possible that it is Thales' own.



Proof: Let a semicircle be drawn with center O and diameter BC. Choose a point A on the semicircle. We must prove that $\widehat{BAC}$ is a right angle. Draw line OA and consider the triangle AOB. Since OB and OA are radii of the semicircle, they have the same length, and so AOB is an isosceles Triangles. Hence the angles $\widehat{ABO}$ and $\widehat{BAO}$ are equal; call them both α. Likewise the angles $\widehat{OAC}$ and $\widehat{OCA}$ are equal; call them both β.

From the large triangle ABC we see that

$$2 \text{ right angles} = \hat{ABC} + \hat{ACB} + \hat{BAC}$$
$$= \alpha + \beta + (\alpha + \beta)$$
$$= 2(\alpha + \beta) .$$

Hence $\hat{BAC} = \alpha + \beta =$ right angle. ■

It has to be noted that the history of the first 300 years of Greek Mathematics (from 600 BC to 300 BC) is obscured by the Greatness of Euclid's elements, written about 300 BC. This work completely eclipsed so many preceding Greek writings on mathematics that those earlier works were therefore discarded and have become lost to us.

The account of the mathematical achievements of Thales was furnished by the Eudemian Summary of Proclus (V th century AD). This summary constitutes the opening pages of Proclus' Commentary on Euclid, Book I.

The next outstanding Greek mathematician mentioned in the Eudemian Summary

is Pythagoras (572-497 BC). He was born on the Aegean island of Samos. Given the proximity of time and place, it may be that Pythagoras studied under Thales. He then appears to have sojourned in Egypt. Returning home, he found Samos under the tyranny of Polycrates and Ionia under the dominations of Persia. Thus he emigrated to Crotone (Southern Italy).

There he gathered around him a group of disciples, later known as the Pythagoreans. The group was both a religious order and a philosophical school.

In time, the influence and aristocratic tendencies of the brotherhood became so great that the democratic forces of Southern Italy destroyed the buildings of the school and caused the society to disperse. Pythagoras fled to Metapontum, where he died (perhaps murdered). The brotherhood, although scattered, continued to exist for at least 2 centuries. Their most outstanding leader was Archytas of Tarentum (400 BC), and to whose school much of the Pythagorean mathematics may be ascribed