MA 361 - 04/20/2012 THIRD MIDTERM (take home)	Spring 2012 A. Corso	Name: _____ _____

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; JUSTIFY YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1.	10	
2.	10	
3.	10	
4.	10	
5.	10	
TOTAL	50	/50

1. Let $\varphi : G \longrightarrow G'$ be a group homomorphism.

Show that if $|G'|$ is finite, then $|\varphi(G)|$ is finite and is a divisor of $|G'|$.
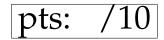
2. Find all left cosets of the subgroup $\{\rho_0, \mu_2\}$ of the group D_4 described by Table 8.12 (on page 80 of our textbook).

pts: /10

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3. Let S be any subset of a group G.

(a) Show that $H_S = \{x \in G \mid xs = sx \text{ for all } s \in S\}$ is a subgroup of G.

(b) In reference to part (a), the subgroup H_G is called the **<u>center of</u>** G.
Show that H_G is an abelian group.

(c) By analyzing Table 8.12 (on page 80 of our textbook), compute the center of the group D_4.

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pts: /10

4. Show that if H is a subgroup of index 2 in a finite group G, then every left coset of H is also a right coset of H.

$\boxed{\text{pts: } /10}$

5. (a) Find the index of $\langle \overline{3} \rangle$ in the group $\mathbb{Z}_{24}$.

(b) Let $\sigma = (1\,2\,5\,4)(2\,3)$ in S_5. Find the index of $\langle \sigma \rangle$ in S_5.

(c) Let H be a subgroup of a group G such that $g^{-1}hg \in H$ for all $g \in G$ and all $h \in H$. Show that every left coset gH is the same as the right coset Hg.

$\boxed{\text{pts:} \quad /10}$