Elementary Modern Algebra I, MA 361 – Spring 2012
Homework set # 1 (Section 0) (due on **January 27** (**Friday**), **2012**)

1. (#4 on page 8) Describe the set $\{m \in \mathbb{Z} \,|\, m^2 - m < 115\}$ by listing its elements.

2. (#5 on page 8) Decide whether the object described

$$\{n \in \mathbb{Z}^+ \,|\, n \text{ is a large number}\}$$

is indeed a set (is well defined). Give an alternative description of each set.

3. (#10 on page 8) Decide whether the object described

$$\{x \in \mathbb{Q} \,|\, x \text{ may be written with a positive denominator less than } 4\}$$

is indeed a set (is well defined). Give an alternative description of each set.

4. (#16 on page 9) List the elements of the power set of the given set and give the cardinality of the power set.

a. $\emptyset$
b. $\{a\}$
c. $\{a, b\}$
d. $\{a, b, c\}$

5. (#17 on page 9) Let A be a finite set, and let $|A| = s$. Based on the preceding exercise, make a conjecture about the value of $|\mathcal{P}(A)|$. Then try to prove your conjecture.

6. (#36 on page 9) Let $n \in \mathbb{Z}^+$ and let $\sim$ be defined on $\mathbb{Z}$ by $r \sim s$ if and only if $r - s$ is divisible by n, that is, if and only if $r - s = nq$ for some $q \in \mathbb{Z}$.

a. Show that $\sim$ is an equivalence relation on $\mathbb{Z}$.
b. Show that, when restricted to the subset $\mathbb{Z}^+$ of $\mathbb{Z}$, this $\sim$ is the equivalence relation, *congruence modulo* n, of Example 0.20.
c. The cells of this partition of $\mathbb{Z}$ are *residue classes modulo* n in $\mathbb{Z}$.