Exercises 1 through 4 concern the binary operation $*$ defined on $S = \{a, b, c, d, e\}$ by means of the table below:

\[
\begin{array}{c|ccccc}
* & a & b & c & d & e \\
\hline
a & a & b & c & b & d \\
b & b & c & a & e & c \\
c & c & a & b & b & a \\
d & b & e & b & e & d \\
e & d & b & a & d & c \\
\end{array}
\]

1. Compute $b * d$, $c * c$, and $[(a * c) * e] * a$.
2. Compute $(a * b) * c$ and $a * (b * c)$. Can you say on the basis of this computations whether $*$ is associative?
3. Compute $(b * d) * c$ and $b * (d * c)$. Can you say on the basis of this computation whether $*$ is associative?
4. Is $*$ commutative? Why?
5. Complete the table below so as to define an associative binary operation $*$ on $S = \{a, b, c, d\}$.

\[
\begin{array}{c|ccc}
* & a & b & c \\
\hline
a & a & b & c \\
b & b & d & c \\
c & c & a & d \\
d & d & & a \\
\end{array}
\]

7. Let $*$ be defined on $\mathbb{Z}$ by letting $a * b = a - b$. Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.
8. Let $*$ be defined on $\mathbb{Q}$ by letting $a * b = ab + 1$. Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.

14. “A binary operation $*$ is **commutative** if and only if $a * b = b * a$.”

Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

15. “A binary operation $*$ on a set $S$ is **associative** if and only if, for all $a, b, c \in S$, we have $(b * c) * a = b * (c * a)$.”

Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

16. “A subset $H$ of a set $S$ is **closed** under a binary operation $*$ on $S$ if and only if $(a * b) \in H$ for all $a, b \in S$.”

Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.
17. On \( \mathbb{Z}^+ \) define \( * \) by letting \( a * b = a - b \). Determine whether the definition of \( * \) does give a binary operation on the set. In the event that \( * \) is not a binary operation, state whether Condition 1, Condition 2, or both of these conditions on page 24 are violated.

23. Let \( H \) be the subset of \( M_2(\mathbb{R}) \) consisting of all matrices of the form \[
\begin{pmatrix}
a & -b \\
b & a
\end{pmatrix}
\] for \( a, b \in \mathbb{R} \). Is \( H \) closed under
   a. matrix addition?
   b. matrix multiplication?

24. Mark each of the following true or false.

   a. If \( * \) is any binary operation on any set \( S \), then \( a * a = a \) for all \( a \in S \).
   b. If \( * \) is any commutative binary operation on any set \( S \), then \( a * (b * c) = (b * c) * a \) for all \( a, b, c \in S \).
   c. If \( * \) is any associative binary operation on any set \( S \), then \( a * (b * c) = (b * c) * a \) for all \( a, b, c \in S \).
   d. The only binary operations of any importance are those defined on sets of numbers.
   e. A binary operation \( * \) on a set \( S \) is commutative if there exist \( a, b \in S \) such that \( a * b = b * a \).
   f. Every binary operation defined on a set having exactly one element is both commutative and associative.
   g. A binary operation on a set \( S \) assigns at least one element of \( S \) to each ordered pair of elements of \( S \).
   h. A binary operation on a set \( S \) assigns at most one element of \( S \) to each ordered pair of elements of \( S \).
   i. A binary operation on a set \( S \) assigns exactly one element of \( S \) to each ordered pair of elements of \( S \).
   j. A binary operation on a set \( S \) assigns more than one element of \( S \) to some ordered pair of elements of \( S \).