Elementary Modern Algebra I, MA 361 – Spring 2012
Homework set # 3 (Section 2) (due on **February 10 (Friday), 2012**)

Exercises 1 through 4 concern the binary operation $*$ defined on $S = \{a, b, c, d, e\}$ by means of the table below:

$*$	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

1. Compute $b * d$, $c * c$, and $[(a * c) * e] * a$.

2. Compute $(a * b) * c$ and $a * (b * c)$. Can you say on the basis of this computations whether $*$ is associative?

3. Compute $(b * d) * c$ and $b * (d * c)$. Can you say on the basis of this computation whether $*$ is associative?

4. Is $*$ commutative? Why?

5. Complete the table below so as to define an associative binary operation $*$ on $S = \{a, b, c, d\}$.

$*$	a	b	c	d
a	a	b	c	
b	b	d		c
c	c	a	d	b
d	d			a

7. Let $*$ be defined on $\mathbb{Z}$ by letting $a*b = a-b$. Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.

8. Let $*$ be defined on $\mathbb{Q}$ by letting $a * b = ab + 1$. Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.

14. "A binary operation $*$ is *commutative* if and only if $a * b = b * a$."
Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

15. "A binary operation $*$ on a set S is *associative* if and only if, for all $a, b, c \in S$, we have $(b * c) * a = b * (c * a)$."
Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

16. "A subset H of a set S is *closed* under a binary operation $*$ on S if and only if $(a * b) \in H$ for all $a, b \in S$."
Correct the definition of the italicized term without reference to the text, if correction is needed, so that it is in a form acceptable for publication.

17. On $\mathbb{Z}^+$ define $*$ by letting $a * b = a - b$. Determine whether the definition of $*$ does give a binary operation on the set. In the event that $*$ is not a binary operation, state whether Condition 1, Condition 2, or both of these conditions on page 24 are violated.

23. Let H be the subset of $M_2(\mathbb{R})$ consisting of all matrices of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ for $a, b \in \mathbb{R}$. Is H closed under

a. matrix addition?

b. matrix multiplication?

24. Mark each of the following true or false.

_____ **a.** If $*$ is any binary operation on any set S, then $a * a = a$ for all $a \in S$.

_____ **b.** If $*$ is any commutative binary operation on any set S, then $a*(b*c) = (b*c)*a$ for all $a, b, c \in S$.

_____ **c.** If $*$ is any associative binary operation on any set S, then $a * (b * c) = (b * c) * a$ for all $a, b, c \in S$.

_____ **d.** The only binary operations of any importance are those defined on sets of numbers.

_____ **e.** A binary operation $*$ on a set S is commutative if there exist $a, b \in S$ such that $a * b = b * a$.

_____ **f.** Every binary operation defined on a set having exactly one element is both commutative and associative.

_____ **g.** A binary operation on a set S assigns at least one element of S to each ordered pair of elements of S.

_____ **h.** A binary operation on a set S assigns at most one element of S to each ordered pair of elements of S.

_____ **i.** A binary operation on a set S assigns exactly one element of S to each ordered pair of elements of S.

_____ **j.** A binary operation on a set S assigns more than one element of S to some ordered pair of elements of S.