Elementary Modern Algebra I, MA 361 – Spring 2012
Homework set # 7 (Section 6) (due on **March 28** (**Wednesday**), **2012**)

An isomorphism of a group with itself is an **automorphism of the group**. In Exercises 1 through 5, find the number of automorphisms of the given group.

1. (#12 on page 66) $\mathbb{Z}_2$

2. (#13 on page 66) $\mathbb{Z}_6$

3. (#14 on page 66) $\mathbb{Z}_8$

4. (#15 on page 66) $\mathbb{Z}$

5. (#16 on page 66) $\mathbb{Z}_{12}$

In Exercises 6 though 8, either give an example of a group with the property described, or explain why no example exists.

6. (#33 on page 67) A finite group that is not cyclic.

7. (#34 on page 67) An infinite group that is not cyclic.

8. (#37 on page 67) A finite cyclic group having four generators.

9. (#45 on page 67) Let r and s be positive integers. Show that $\{nr + ms \,|\, n, m \in \mathbb{Z}\}$ is a subgroup of $\mathbb{Z}$.

10. (#46 on page 67) Let a and b be elements of a group G. Show that if ab has finite order n, then ba also has order n.

11. (#47 on page 67) Let r and s be positive integers.
- **a.** Define the **least common multiple** of r and s as a generator of a certain cyclic group.
- **b.** Under what condition is the least common multiple of r and s their product, rs?
- **c.** Generalizing part **b.**, show that the product of the greatest common divisor and the least common multiple of r and s is rs.

12. (#49 on page 67) Show by a counterexample that the following "converse" of Theorem 6.6 is not a theorem: "If a group G is such that every subgroup is cyclic, then G is cyclic."

13. (#50 on page 67) Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and is the *unique* such element. Show that $ax = xa$ for all $x \in G$. [*Hint:* Consider $(xax^{-1})^2$.]

14. (#51 on page 67) Let p and q be distinct prime numbers. Find the number of generators of the cyclic group $\mathbb{Z}_{pq}$.