Section 8

Homework Assignment

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$, $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$

and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$ be permutations

in S_6.

#1. Compute $\tau\sigma$.

Ans: $\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 6 & 5 & 4 \end{pmatrix}$.

#2. Compute $\tau^2\sigma$.

Ans: $\tau^2\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$.

#3. Compute $\mu\sigma^2$.

Ans: $\mu\sigma^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 6 & 2 & 5 \end{pmatrix}$.

#4. Compute $\sigma^{-2}\tau$.

Ans: $\sigma^{-2}\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 6 & 2 & 4 & 3 \end{pmatrix}$.

#5. Compute $\sigma^{-1}\tau\sigma$.

Ans. $\sigma^{-1}\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 5 & 4 & 3 \end{pmatrix}$.

#6. Compute $|\langle\sigma\rangle| = 6$.

Ans: Since $\sigma^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} = id$,

We conclude that $|\langle\sigma\rangle| = 6$.

#8. Compute σ^{100}.

Ans: Since $100 = 16\cdot 6 + 4$ one has

$$\sigma^{100} = \sigma^{6\cdot 16 + 4} = (\underbrace{\sigma^6}_{id})^{16}\cdot \sigma^4 = \sigma^4$$

$$\therefore\ \sigma^{100} = \sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 3 & 4 \end{pmatrix}.$$

#11. Let A be a set and let $\sigma \in S_A$.
For a fixed $a \in A$, the set
$\mathcal{O}_{a,\sigma} = \{\sigma^n(a) \mid n \in \mathbb{Z}\}$
is the orbit of a under σ.

Find the orbit of 1 under the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \in S_6$.

Ans:

$$\mathcal{O}_{1,\sigma} = \{1, 3, 4, 5, 6, 2\} = \{1, 2, 3, 4, 5, 6\}$$

$\therefore$ there is just one orbit.

#12. Find the orbit of 1 under the permutation $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix} \in S_6$.

Ans:

$O_{1,\tau} = \{1, 2, 4, 3\}$.

#17. Find the number of elements in the set $\{\sigma \in S_5 \mid \sigma(2) = 5\}$.

Ans: Notice that we have 4 choices for $\sigma(1)$; we have 3 choices for $\sigma(3)$; we have 2 choices for $\sigma(4)$ and one choice for $\sigma(5)$.

Overall, we have $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$ choices.

#20. Give the multiplication table for the cyclic subgroup of S_5 generated by

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}.$$

There will be six elements. Let them be $\rho, \rho^2, \rho^3, \rho^4, \rho^5$, and $\rho^0 = \rho^6$. Is this group isomorphic to S_3?

Ans: You can check that $\rho^6 = id = \rho^0$.

Thus $\langle \rho \rangle = \{\rho^0, \rho^1, \rho^2, \rho^3, \rho^4, \rho^5\}$ is an abelian group with 6 elements. It cannot be isomorphic to S_3 as S_3 is not abelian.

The multiplication table for $\langle \rho \rangle$ is

$\cdot$	ρ^0	ρ	ρ^2	ρ^3	ρ^4	ρ^5
ρ^0	ρ^0	ρ	ρ^2	ρ^3	ρ^4	ρ^5
ρ	ρ	ρ^2	ρ^3	ρ^4	ρ^5	ρ^0
ρ^2	ρ^2	ρ^3	ρ^4	ρ^5	ρ^0	ρ
ρ^3	ρ^3	ρ^4	ρ^5	ρ^0	ρ	ρ^2
ρ^4	ρ^4	ρ^5	ρ^0	ρ	ρ^2	ρ^3
ρ^5	ρ^5	ρ^0	ρ	ρ^2	ρ^3	ρ^4

Actually, $\langle \rho \rangle$ is isomorphic to $\mathbb{Z}_6$.