Linear Algebra, MA 565 – Fall 2011
Homework set # 1 (due on **September 7 (Wednesday), 2011**)

1. (#4 on page 39) Let W be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in $\mathbb{R}^5$ which satisfy

$$\begin{cases} 2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \\ x_1 + \frac{2}{3}x_3 - x_5 = 0 \\ 9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0 \end{cases}$$

Find a finite set of vectors which spans W.

2. (#5 on page 40) Let F be a field and let n be a positive integer ($n \geq 2$). Let V be the vector space of all $n \times n$ matrices over F. Which of the following sets of matrices A in V are subspaces of V?
(a) all invertible A;
(b) all non-invertible A;
(c) all A such that $AB = BA$, where B is some fixed matrix in V;
(d) all A such that $A^2 = A$.

3. (#7 on page 40) Let W_1 and W_2 be subspaces of a vector space V such that the set-theoretic union of W_1 and W_2 is also a subspace. Prove that one of the spaces W_i is contained in the other.

4. (#8 on page 40) Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let V_e be the subset of even functions, $f(-x) = f(x)$; let V_o be the subset of odd functions, $f(-x) = -f(x)$.
(a) Prove that V_e and V_o are subspaces of V.
(b) Prove that $V_e + V_o = V$.
(c) Prove that $V_e \cap V_o = \{0\}$.