Linear Algebra, MA 565 – Fall 2011

Homework set # 10 (due on **December 7 (Wednesday), 2011**)

40. (*a*) **(6.2. #8)** Let A and B be $n \times n$ matrices over the field F. Prove that if $I_n - AB$ is invertible, then $I_n - BA$ is invertible and

$$(I_n - BA)^{-1} = I_n + B(I_n - AB)^{-1}A.$$

(*b*) **(6.2. #9)** Prove that if A and B are $n \times n$ matrices over F, then AB and BA have precisely the same eigenvalues in F. (You might want to use part (*a*).)

41. (6.2. #13) Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$ which are continuous. Let T be the linear operator on V defined by

$$(Tf)(x) = \int_0^x f(t)\,dt.$$

Prove that T has no eigenvalues.

42. (6.3. #10) Let V be the vector space of $n \times n$ matrices over the field F. Let A be a fixed $n \times n$ matrix. Let T be the linear operator on V defined by

$$T(B) = AB.$$

Show that the minimal polynomial for T is the minimal polynomial for A.

43. (6.4. #4) Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0.5 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{bmatrix}$.

Are A and/or B similar over $\mathbb{R}$ to a triangular matrix? If so, find such a triangular matrix.

44. (6.4. #5) Every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

45. (6.4. #10) Let A be a 3×3 matrix with real entries. Prove that, if A is not similar over $\mathbb{R}$ to a triangular matrix, then A is similar over $\mathbb{C}$ to a diagonal matrix.