Linear Algebra, MA 565 – Fall 2011
Homework set # 2 (due on **September 14** (**Wednesday**), **2011**)

5. Decide the dependence or independence of:
(a) $\mathbf{v}_1 - \mathbf{v}_2$, $\mathbf{v}_2 - \mathbf{v}_3$, $\mathbf{v}_3 - \mathbf{v}_4$, and $\mathbf{v}_4 - \mathbf{v}_1$ for any vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$;
(b) $(1, 1, 2)$, $(1, 2, 1)$ and $(3, 1, 1)$.

6. In the vector space V of all polynomials $P(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$ of degree up to three, let W be the subset of all polynomials with

$$\int_0^1 P(x)\, dx = 0.$$

Verify that W is a subspace of V and find a basis for W.

7. (#6 & 7 on pages 48-9) Let V be the vector space of all 2×2 matrices over a field F. Prove that V has dimension 4 by exihibiting a basis for V which has four elements. Then, let W_1 be the set of matrices of the form

$$\begin{bmatrix} x & -x \\ y & z \end{bmatrix} \qquad x, y, z \in F$$

and let W_2 be the set of matrices of the form

$$\begin{bmatrix} a & b \\ -a & c \end{bmatrix} \qquad a, b, c \in F.$$

(a) Prove that W_1 and W_2 are subspaces of V.
(b) Find the dimensions of W_1, W_2, $W_1 + W_2$, and $W_1 \cap W_2$ and verify that the formula $\dim W_1 + \dim W_2 = \dim(W_1 + W_2) + \dim(W_1 \cap W_2)$ holds in this specific example.

8. (#10 on page 49) Let V be a vector space over the field F. Suppose that there are a finite number of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_r$ in V which span V. Prove that V is finite-dimensional (i.e., V has a finite basis).