Linear Algebra, MA 565 – Fall 2011
Homework set # 3 (due on **September 21 (Wednesday), 2011**)

9. (#7 on page 73) Let F be a subfield of the complex numbers and let T be the function from F^3 into F^3 defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$$

(a) Verify that T is a linear transformation.

(b) If (a, b, c) is a vector in F^3, what are the conditions on a, b, and c that the vector be in the range of T? What is the rank of T?

(c) What are the conditions on a, b, and c that (a, b, c) be in the null space of T? What is the nullity of T?

10. (#9 on page 73) Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$, verify that T is a linear transformation from V into V.

11. (#12 on page 74) Let V be an n-dimensional vector space over the field F and let T be a linear transformation from V into V such that the range and null space of T are identical. Prove that n is even. Provide an example of such a linear transformation.

12. (#13 on page 74) Let V be a vector space and T a linear transformation from V into V. Prove that the following statements about T are equivalent:

(a) The intersection of the range of T and the null space of T is the zero subspace of V.

(b) If $T(T(\mathbf{v})) = \mathbf{0}$, then $T(\mathbf{v}) = \mathbf{0}$.