Linear Algebra, MA 565 – Fall 2011
Homework set # 6 (due on **October 19** (**Wednesday**), **2011**)

22. (Exercise 5 – Leep) Using our usual notation, $f \in \mathcal{L}(V, W)$ is an isomorphism if and only if $[f]_\beta^\gamma$ is an invertible matrix.

23. (Exercise 6 – Leep) Let $A \in \mathcal{M}_{m\times n}(F)$ and let $B \in \mathcal{M}_{n\times m}(F)$, where F is a field.
(a) Suppose that $AB = I_m$. Then $m \leq n$.
(b) Suppose that $AB = I_m$ and $m = n$. Then $BA = I_m$. Thus, if $n = m$, then $AB = I_m$ if and only if $BA = I_m$.
(c) Suppose that $AB = I_m$ and $m < n$. Then $BA \neq I_n$.

24. (Exercise 11 – Leep) Let $\beta = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ be a basis of V and let $\gamma = \{\mathbf{w}_1, \dots, \mathbf{w}_m\}$ be a basis of W. Set $V_j = \langle \mathbf{v}_j \rangle$ and $W_j = \langle \mathbf{w}_j \rangle$ so that $V = \bigoplus_{j=1}^{n} V_j$ and $W = \bigoplus_{i=1}^{m} W_i$. Let $f \in \mathcal{L}(V, W)$ and suppose that $[f]_\beta^\gamma = (a_{ij})$. Then show that

$$f = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}$$

where $f_{ij} \in \mathcal{L}(V_j, W_i)$ and $f_{ij}(\mathbf{v}_j) = a_{ij}\mathbf{w}_i$.

25. (# 8 on page 96) Let θ be a real number. Prove that the following two matrices are similar over the field of complex numbers:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix}.$$

(**Hint:** Let T be the linear operator on $\mathbb{C}^2$ which is represented by the first matrix in the standard ordered basis. Then find vectors $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ such that $T\boldsymbol{\alpha}_1 = e^{i\theta}\boldsymbol{\alpha}_1$, $T\boldsymbol{\alpha}_2 = e^{-i\theta}\boldsymbol{\alpha}_2$, and $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2\}$ is a basis.)

26. Read and understand Examples 15 and 17 from the book of Hoffman and Kunze (pages 89-94 of the textbook).