Linear Algebra, MA 565 – Fall 2011
Homework set # 7 (due on **October 26** (**Wednesday**), **2011**)

27. (Exercise 8 – Leep) Prove the three statements in Proposition 3.21. That is, let $A \in M_{m\times n}(k)$ and let $A_1 = E_{ij}(a)A$, $A_2 = D_i(a)A$ and $A_3 = P_{ij}A$, where the elementary matrices lie in $M_{m\times m}(k)$. Then

- A_1 is obtained from A by adding a times row j of A to row i of A.
- A_2 is obtained from A by multiplying row i of A by a.
- A_3 is obtained from A by interchanging rows i and j of A.

28 Let S_4 denote the set of all permutations of four objects. This is a group under composition of functions. Consider the permutations $\sigma_1 = (2\ 3\ 4)$ and $\sigma_2 = (1\ 2\ 3)$. Write the permutation matrices P_{σ_1} and P_{σ_2} associated to σ_1 and σ_2, respectively. Compute $\sigma_2 \circ \sigma_1$ and $P_{\sigma_2\circ\sigma_1}$. What is its relation with P_{σ_1} and P_{σ_2}?

29 Consider the 3×4 matrix A given below

$$A = \begin{bmatrix} 1 & 1 & -2 & -1 \\ 2 & 2 & -3 & -3 \\ 3 & 3 & -5 & -4 \end{bmatrix}$$

Find invertible matrices $D \in M_{3\times 3}(k)$ and $E \in M_{4\times 4}(k)$ such that DAE has the form described in Proposition 3.32. What is the rank r of A?
(Feel free to use Matlab for your calculations, if you like.)