Review Problems for the Final Exam/MA 213, Fall 2013

(Exam time: Monday, December 16, 10:30 am-12:30 pm, CB 106)

The final exam is comprehensive and covers Chapters 12.1-12.6, 13.1-13.5, 14.1-14.7, 15.1-15.6, 16.1-16.4, and Chapter 17.1-17.2. The review problem sheets for the two midterm exams and the following review problem for Chapters 16.1-16.4, 17.1-17.2 can be viewed as the whole review set for the final exam.

• Review the definition formula for calculating curl $\mathbf{F}$, where $\mathbf{F}$ is a vector field in $\mathbb{R}^2$ or $\mathbb{R}^3$.

• Review the statements for Green’s Theorem, Stokes’ Theorem.

• Review parametric equations for surface, and the formula for its normal vector, and surface area elements.

• Review the definition formula for line integrals, surface areas, and surface integrals.

Review problems from Chapter 16 and Chapter 17

1. Evaluate $\int_C x \sin y \, dx + xyz \, dz$, where $C$ is given by $x = t, y = t^2, z = t^3$ for $0 \leq t \leq 1$.

2. Evaluate $\int_C x^3 y \, dx - x \, dy$, where $C$ is the circle $\{(x, y)|x^2 + y^2 = 1\}$ with counterclockwise orientation.

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y) = (x^2 y, e^y)$, $C = \{(t^2, -t^3)|0 \leq t \leq 1\}$.

4. Use the Green’s theorem to evaluate $\int_C x^2 y \, dx - y^2 \, dy$, where $C$ is $\{(x, y)|x^2 + y^2 = 4\}$ with counterclockwise orientation.

5. Use the Green’s theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where

$$\mathbf{F}(x, y) = \left( y - \ln(x^2 + y^2), \ 2 \tan^{-1}\left( \frac{y}{x} \right) \right)$$

and $C$ is the circle $(x - 2)^2 + (y - 3)^2 = 1$ oriented counterclockwise.
6. Calculate curl of the vector fields

\[ \mathbf{F}(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}(x, y, z), \]

and

\[ \mathbf{G}(x, y, z) = (y^2 z^3, 2xyz^3, 3xy^2 z^2). \]

7. Determine whether or not the vector field is conservative. If it is conservative, find a function \( f \) such that \( \mathbf{F} = \nabla f \).

\[ \mathbf{F}(x, y, z) = \langle y \cos(xy), x \cos(xy), -\sin z \rangle, \]

and

\[ \mathbf{F}(x, y, z) = \langle 2xy, (x^2 + 2yz), y^2 \rangle. \]

8. Calculate the area of the surface

\[ z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}}), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \]

9. Calculate the area of the surface with parametric equations \( x = u^2, \ y = uv, \ z = \frac{1}{2}v^2, \ 0 \leq u \leq 1, \ 0 \leq v \leq 2. \)

10. Calculate \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle \) and \( S \) is the part of the paraboloid \( z = 4 - x^2 - y^2 \) that lies above the square \( 0 \leq x \leq 1, 0 \leq y \leq 1, \) and has upward orientation.

11. Use Stokes’ Theorem to evaluate \( \int_C \mathbf{F} \cdot d\mathbf{s} \), where \( \mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, z + x^3 \rangle \) and \( C \) is the triangle with vertices \((1, 0, 0), (0, 1, 0), \) and \((0, 0, 1)\).

12. Use Stokes’ Theorem to evaluate \( \int \int_S \text{curl} \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F}(x, y, z) = (2y \cos z, e^z \sin z, xe^y) \) and \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 9, \ z \geq 0, \) oriented upward.