On the geometry of the p-harmonic world

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The main subject of the presentation is the geometry of p-harmonic mappings. These are $W^{1,p}_{loc}(\Omega, \mathbb{R}^n)$ solutions to the so called p-harmonic system:

$$\operatorname{div}(|Du|^{p-2}Du) = 0, \quad u = (u^1, \dots, u^n) : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n, \quad 1 < p < \infty$$

where Du is the Jacobi matrix. We will explain motivation for studying such mappings and discuss some classes of p-harmonics.

The classical result by Bojarski and Iwaniec shows that the complex gradient of the planar p-harmonic function is a quasiregular mapping. We extend this result to the setting of p-harmonic mappings in the plane. Namely, to every p-harmonic mapping in the plane with $p \geqslant 2$ there corresponds a quasilinear system of first order PDE's which couples the complex gradients of the coordinate functions of the mapping. We will discuss the ellipticity of such system and describe the relation between planar quasiregular mappings and p-harmonic mappings. The radial p-harmonic mappings will serve as a illustration of our results. The p-harmonic conjugate problem will be stated.

Despite similarity of the definition to the scalar counterpart, the setting of mappings turns out to be much more difficult and challenging. Many questions remain open, we will address some of them.