The Dirichlet Eigenvalue Problem for the Lamé System and for Elliptic Systems on Perturbed Domains

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Abstract

We consider eigenvalues of an elliptic operator

$$(Lu)^\beta = -\frac{\partial}{\partial x_j}\left(a_{ij}^{\alpha\beta}\frac{\partial u^\alpha}{\partial x_i}\right) \qquad \beta = 1, ..., m$$

where $u = (u^1, ..., u^m)^t$ is a vector valued function and $a^{\alpha\beta}(x)$ are $(n \times n)$ matrices whose elements $a_{ij}^{\alpha\beta}(x)$ are uniformly bounded measurable functions such that

$$a_{ij}^{\alpha\beta}(x) = a_{ji}^{\beta\alpha}(x)$$

for any combination of α, β, i, and j. If we have two non-empty, open, disjoint, and bounded sets, Ω and $\widetilde{\Omega}$, in $\mathbb{R}^n$, and add a set T_ε of small measure to form the domain $\Omega_\varepsilon = \Omega \cup \widetilde{\Omega} \cup T_\varepsilon$, then we show that as $\varepsilon \to 0^+$, the Dirichlet eigenvalues corresponding to the family of domains $\{\Omega_\varepsilon\}_{\varepsilon>0}$ converge to the Dirichlet eigenvalues corresponding to $\Omega_0 = \Omega \cup \widetilde{\Omega}$. In this paper, we consider systems which satisfy either a strong ellipticity condition or, in particular, the Lamé system.