Review Problems for Final Exam, MA 214, Spring 2012

- The final exam covers Chapter 2, Chapter 3, Chapter 4, Chapter 5.1, 5.2, and Chapter 6.
- It will take place Monday April 30, 8-10 am at CB 339.
- It will consist of 10 problems.

The following is a list of review problems that may help you to prepare for the final exam.

(1). Use the integral factor method to solve the initial value problem

\[(1 + t^2)y' + 4ty = (1 + t^2)^{-2}\]
\[y(0) = 1\]

(2). Solve the separable equation

\[y^2(1 - x^2)^{\frac{1}{2}}dy = \arcsin xdx\]
\[y(0) = 1\]

(3). Verify that the following equation is exact. Then solve it.

\[(ye^{xy} \cos 2x - 2e^{xy} \sin 2x + 2x) dx + (xe^{xy} \cos 2x - 3) dy = 0.\]

(4). Suppose that a certain population obeys the logistic equation

\[\frac{dy}{dt} = ry[1 - (y/K)]\]

(a) If \(y_0 = \frac{K}{2}\), find the time \(\tau\) at which the initial population has doubled. Find the value of \(\tau\) corresponding to \(r = 0.025\) per year.

(b) If \(y_0/K = \alpha\), find the time \(T\) at which \(y(T)/K = \beta\), where \(0 < \alpha, \beta < 1\). Observe that \(T \to \infty\) as \(\alpha \to 0\) or as \(\beta \to 1\). Find the value of \(T\) for \(r = 0.025\) per year, \(\alpha = 0.1\), and \(\beta = 0.9\).

(5). State the definition of linear dependence(or independence) of \(n\)-functions: \(f_1(t), \cdots, f_n(t)\). Use the Wronskian method to check that \(y_1(t) = e^{\lambda_1 t}\),
$y_2(t) = e^{\lambda_2 t}$, and $y_3(t) = e^{\lambda_3 t}$ are linearly independent, provided that $\lambda_1, \lambda_2, \lambda_3$ are distinct real values.

(6). Use the characteristic method to find a general solution to the following equation:

$$y^{(8)} - y = 0.$$  

(7). Use the Laplace transformation method to solve the initial value problem for

$$y^{(8)} - 3y'' + 2y' = t + e^t$$

$$y(0) = 1$$

$$y'(0) = -1$$

$$y''(0) = -\frac{3}{2}. $$

(8). Use the underdetermined coefficient method to find the right form of a particular solution to

$$y^{(4)} + 2y^{(3)} + 2y'' = 3e^t + te^{-t} + e^{-t}\sin t.$$  

(9). Use the variation of parameter method to find a formula involving integrals for a particular solution of the differential equation

$$y'' - y'' + y' - y = g(t)$$

(10). Find a series solution of the equation

$$y'' + xy' + 5y = 0, \ y(0) = 1, y'(0) = -1.$$  

(11). Find the first five terms in the series solution of the following initial value problem

$$(1 + x)y'' - xy' + 4x^2 y = 0, \ y(0) = 1, \ y'(0) = 3.$$ 

(12). Find the Laplace transform of the following functions

$$e^{2t}\sinh(3t), \ u_1(t)t^n, \ t^2 e^t \cos t.$$
(13). Find the inverse Laplace transform of the following functions by the partial fraction method

\[
\frac{8s^2 - 4s + 12}{(s - 1)(s + 2)(s^2 + 1)}, \quad \frac{2s + 5}{s^2 + 2s + 17}.
\]

(14). Find the solution of the initial value problem by the Laplace transform

\[y'' + y' + \frac{5}{4}y = 1 - u_\pi(t) \sin t, \quad y(0) = 1, \quad y'(0) = -1.\]

(15). Find the solution of the initial value problem by the Laplace transform

\[y'' + 4y = g, \quad y(0) = 0, \quad y'(0) = 0,\]

where

\[g(t) = 0, \quad 0 < t < 5; = \frac{t - 5}{5}, \quad 5 \leq t < 10; = 1, \quad t \geq 10.\]