

The New Bernoulli Edition

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# The New Bernoulli Edition

## By C. Truesdell \*

WHAT did the Bernoullis give to mathematics and mechanics? To answer this question, look in the standard histories. You will find tags such as "brachistochrone" and a few odd details, mostly trivial, associated with the names of James I, John I, and Daniel, besides some generalities to the effect that they were great men. For their personal relations, everyone has heard vaguely of the quarrel between the great brothers, but even a best-seller featuring gossip about mathematicians gives us beyond this only its author's musings on the Bernoulli family as material for a study of heredity and environment. No considerable biography of any Bernoulli has ever been published.

Going to the shelves of any good mathematical library so as to form your own judgement from the sources, you will find the collected works of every Victorian with a name: Smith, Borchardt, Cremona, Fuchs, Halphen, Hermite, Schwarz, Teixeira, Steiner, etc., but unless the library is exceptional, you will find *nothing* by the Bernoullis. John I Bernoulli supervised the publication of his *Opera Omnia* in four volumes in 1743, the works of James I were published in two volumes in 1744, while the works of Daniel never have been collected. Few libraries have the journals in which the Bernoullis published: The Leipzig *Acta Eruditorum* and the organs of the academies of Paris, Petersburg, and Berlin in the eighteenth century. In fact, in a typical working mathematical library in the U. S. A. there will be found *not a single paper or book* by any Bernoulli. A library now wishing to obtain the journals and the two sets of *Opera* will do so only at great cost and after years of delay.

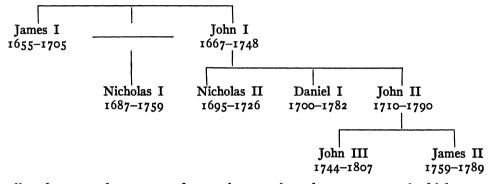
It is thus plain that mathematicians and historians of science are so little interested in the Bernoullis that not only is no modern description available, but even the material from which to draw an analysis is to be found only in the largest libraries. The volume to be discussed below, despite its importance, was given but cursory notice in the two principal organs of review for mathematics in the U. S. A., organs which not long before had published more or less extensive summaries of works on how Babylonians added fractions and of "new" texts on complex variables.

In view of this evident lack of interest, then, little space should be taken up here. I am compelled, however, as fully as within my power, to employ the privilege of commenting upon what seems to me the most important event in the history of mathematics in a quarter of a century. This event is the publication of the first volume of the great new edition of the collected works of the mathematicians Bernoulli.<sup>1</sup>

#### I. PLAN OF THE EDITION

This edition will collect the letters, diaries, unpublished manuscripts, and printed works of the three great Bernoullis, of Nicholas I and II, of John II and III, and of James II. Naturally enough it will include also the literary remains of James Hermann, hardly a compelling figure, but one who takes on special interest as a pupil of the deep and enigmatic James I Bernoulli. The

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editor has traced over 7500 letters from and to these persons, of which over 2300 involve John I and nearly 3000, John III. This brings us at once to the decision faced by the editor. To publish this entire mass would have set the lesser Bernoullis in dominance. The present plan is to publish everything left by the three great Bernoullis, but only a selection from the five lesser ones and Hermann. The edition, then, will be dominated by John I, whose existing letters connect him with over 100 correspondents, including every great scientist of the day except Huygens.

When it comes to works published before, both papers and books, we find only some 450 from the whole group, making about 14,000 typewritten pages. Again John III is the most prolific, and most of his product is non-mathematical. The three great Bernoullis issued about 100 works apiece, 7000 pages in all. Here, as well as in manuscripts, lectures, etc., the editor has decided to print only a small fraction of the output of the lesser figures, taking care to include anything of possible mathematical interest. His summary of the material to be published is shown in the table below, where the numbers indicate pages of typescript.

	Previously printed works				Letters		Biographies	Totals
		Mathematical manuscripts Courses		Lectures	Unprinted	Previously printed		
James I	1300	650	40	30	50	130	200	2400
John I	2100	100	40	280	6480	1230	70	10300
Daniel	2300	100	100	100	920	280		3800
Nicholas I	230	250	10		1400	90	20	2000
Nicholas II	170				70	••	••	240
John II	200	10	• •	50	1000	••	40	1300
James II	200	10			20		30	260
John III	200			40	160		1200	1600
James Hermann	1300		80	120	400	200	••	2100
Total	8000	1120	270	620	10500	1930	1560	24000

TABLE

Except for the correspondence with Leibniz and Euler, which is left for the collected editions of those scientists, the above table is an estimate of the material which this edition is to print. One half consists in letters, one third in scientific productions. Three quarters will be devoted to the preservation of everything concerning the three great Bernoullis, and of this about two thirds

<sup>1</sup>Otto Spiess (editor): Der Briefwechsel von Johann Bernoulli, Herausgegeben von der Naturforschenden Gesellschaft in Basel,

el Band I. 532 pp., 5 plates, 128 line sketches in er facsimile. Basel: Birkhäuser Verlag, 1955, 60 el, Swiss Fr. for John I; the remainder, a selection from the works of the six lesser savants. This makes a planned edition of 20-25 volumes of 700 printed pages each.

The format and type are smaller than for the great editions of earlier years, but the printing and style are excellent. The cost is low for a work of this magnitude, but the binding is not sturdy enough for a book of permanent value. The present volume contains dozens of facsimiles of the small original line sketches as well as five full page plates with portraits of l'Hôpital and Pieter Burman and specimens of the handwritings of John I, of l'Hôpital, and of his wife.

The edition was started officially in 1935 with the creation of the Bernoulli Commission under the patronage of the Basler Naturforschende Gesellschaft. The general editor was and is Otto Spiess, of whom more below. The financing was originally through small gifts from Swiss industries, and now there appears a prospect of support from the new Swiss National Fund for Scientific Research. Even if the financial side is in order, the general editor tells us, it is difficult to find competent and willing editors for the individual volumes.

On pages 77-78, Spiess discusses the relative merits of two possible plans: the *individual*, organized about the persons, and the *collective*, organized about the periods. While he favors the latter as more instructive to the reader, he judges it to be impossible without a more secure financial backing than is assured for the Bernoulli edition. The individual plan is adopted because it more easily allows the material to be divided among numerous editors whose labor can be extended over many years.

- Series 1. Correspondence of John I, in six volumes, followed by one volume apiece for the letters of Daniel, John II, and Nicholas I.
- Series 2. Collected works of the two older Bernoullis and of Nicholas I and Nicholas II, followed by two volumes for the letters and works of Hermann.
- Series 3. Works of Daniel and of John II and his sons, in three volumes.

There is to be a final volume with biography, iconography, and index. The present volume is the first of Series I. Its contents are such as to serve equally well as a unit for 1690-1705 in an edition organized on the collective plan, which may not be out of the question now that the financial prospect is more favorable than when the work was started.

The editor compares the project with the Euler edition, which, as he reminds us, is but half complete after forty years of steady work. The Bernoulli edition will contain about two fifths as much material as the Euler edition.

In respect to critical apparatus, the present volume stands midway between the elaborate commentary of the Huygens edition and the bare text of the Euler edition. The correspondence of John I with James I, with l'Hôpital, and with miscellaneous persons between 1693 and 1706 are presented separately, each with a special introduction by Spiess. From a few lines to a half of each page of text is occupied by footnotes. These are most helpful, being sufficient to make the material readable by any patient person who understands calculus and geometry. Sometimes there is a translation of an argument into modern notation; sometimes a missing proof is supplied; the problems discussed in the letters are indexed by numbers and cross-referenced in the notes, which give full citation of relevant printed works. Missing letters which can be identified are given interpolated numbers, and all known information regarding their content is collected. Otto Spiess has devoted twenty years to the organization of the edition, to the collection of materials for it, and to the editing of this volume. He is now seventy-eight years old. His first product sets a new standard for the editing of mathematical works. Never have I seen an edition so carefully and completely yet unobtrusively and compactly organized, annotated, and crossindexed. Spiess's name appears only at the end of his general preface, not on the title page. Not only is such modesty excessive, but it seems to me that for having realized the inception of this great undertaking and for having given to us such important material in so beautiful and useful a form, Spiess deserves some special tribute from those who cultivate or observe the history of science. I take this occasion to express my heartiest thanks for the generous assistance Spiess gave to my labor in the midst of his own, when he took the pains to go through enormous files of unpublished material so as to select and send to me copies of everything relevant to the Bernoullis' work on the mechanics of fluids and deformable solids.

#### **II.** Spiess's Introduction

The general introduction, 85 pages long, explains not only the organization, principles, and plan of the edition but also the history of the manuscripts and of the search for additional material. It is shown that the existing letters make only a small fraction of what was actually written, and reasons for the loss, as well as conjectures regarding its nature, are put forward.

What is especially interesting and instructive is the fate of the most important existing collection, the great mass of John I's letters. For reasons that will be apparent to the reader of Part IV of this review, it had been John I's plan to publish them in part in his Opera Omnia, and he sent hundreds of them to Cramer, the editor. Of these, the 150 letters to and from Leibniz were published in 1745, but the publisher's and editor's means and energy were insufficient for the rest. Eventually the grandsons of John I inherited the manuscripts. In 1790 John III, who revered his family's tradition and showed responsibility toward it but was in reduced circumstances with eleven children to rear, attempted to sell the books and the manuscripts. First he sought a prince or a large library ready to buy the material and publish the letters. Failing this, he tried to induce a press to buy and publish them. It should be added that the prices he asked were most moderate, and that he was ready to adjust them downward in proportion to the amount of material the buyer was willing to publish. All his attempts to secure publication of any part failed. In two lectures, printed in 1803 and 1804 in the *Histoire* of the languishing academy of Berlin, he disclosed that most of the material had been sent to "an illustrious academy of the North," some copies and minor pieces to "the most grand and rich library of a sovereign prince."

Within a few years, the traces of these two great deposits were obliterated. In a sense, the material was never lost. It stayed, safe and untouched, in the libraries of Gotha and the Stockholm Academy. Pages 32-46 of Spiess's introduction detail the attempts, most futile, of several persons, from the efforts of Rudolf Wolf in the 1840's down to his own in the 1930's, to locate the missing manuscripts. At various times various persons did indeed know of one or the other collection. Nevertheless, internationally published appeals from those seeking information were fruitless, In particular, Eneström appears in Spiess's account as a sort of villain who, it seems, deliberately concealed the

existence of the rediscovered Stockholm collection after the Basler Naturforschende Gesellschaft had refused to finance a proposed edition under his direction in 1887. In 1884 Enestrom had founded the Bibliotheca Mathematica. which stood under his editorship for thirty years as the principal journal of research in the history of mathematics. Here, over two decades, he spread out small extracts or sections of the Bernoulli correspondence, mainly with Euler. but apart from a few brief annotations prior to 1000 he gave no notice of the enormous mass of unpublished material at his disposal. Spiess, explaining that "he who seeks in the letters of savants of the eighteenth century for new scientific material not in the printed works of the correspondents will generally be disappointed," suggests that Eneström "had no organ for the particular charm afforded by the personal testimony of great men." It would be misleading, however, to dismiss the contents of the Bernoulli letters as personal in the narrower sense. Indeed, in my opinion some of the older work on the history of mathematics by confining attention to dry recital of the dates of minutiae has stifled the interest creative mathematicians of today would naturally feel for the creative processes of their predecessors. Even were it desirable, it would be impossible to explain the tides of thought apart from the pull of the thinkers. In the Age of Reason, personal letters and diaries often disclose the mind and its alternatives more clearly than do the published records, sometimes intended to conceal the origins of results or to avoid antagonising particular readers.

In any case, the story told by Spiess shows clearly that during the years when historians of mathematics were most eagerly searching the works of those who in rare examples used ideas of calculus without knowing it, *no one cared* for the splendid flourishing of mathematics at the hands of those whose special pride was the conscious cultivation of infinitesimal methods.

### III. THE CORRESPONDENCE OF JAMES I AND JOHN I BERNOULLI

The famous open letters from the days of the great quarrel are not reprinted. All that remains are four letters from John to James in 1691, with a partial reconstruction of sixteen more. Here we see John still in his student days, growing restive and beginning to show his temper and his rebellious independence from his senior brother. The subject is the *velaria*, the curve assumed by a cylindrical sail under certain hypotheses. The correspondence ends just before each brother discovered independently that this curve is a catenary. The letters show rivalry, suspicion, and misunderstanding. The tone taken by John seems most rude from the start, but it is possible that even 250 years ago this was intimacy permissible between brothers and had to be read with an appropriate tone of voice. A specimen from the third letter follows:

It is you who wish to defy the world, do you think me blind? If I had the Acta I would show you the place. Would it be possible that I could have by heart for all these years this series giving the quadrature of conic sections, if you had just now invented it? And more, what shows your plagiarism is that you say . . . [while] I say that this is false in general. . . .

The fourth letter, although it contains passages in the same tone, ends with John's imploring his older brother to recommend him for the professorship at Groningen which in fact he was soon to obtain.

#### IV. THE CORRESPONDENCE BETWEEN JOHN I BERNOULLI AND L'HÔPITAL

Calculus was made known to the learned world by the brilliant papers of Leibniz and the brothers Bernoulli from 1690 on. The first textbook of the new science appeared in 1696: The Analyse des infiniment petits, pour intelligence des lignes courbes, a work of some 200 pages, issued anonymously but known to be by the Marquis de l'Hôpital (1661-1704). This book went through several editions and remained a standard for a century. Until the present day it has been considered as a work in large part original, and its famous rule on indeterminate forms is known as l'Hôpital's. Beyond this book and a posthumous treatise on conic sections, l'Hôpital's works consist in some twenty-five very short notes on special problems. He had become well known through his sixth publication, a four-page note of 1692, giving the solution of de Beaune's problem of the inverse tangent, which had been outstanding for fifty years.

In letters, some of which have been in print for two centuries, John Bernoulli complained to Leibniz, Varignon, and others that most of what was attributed to l'Hôpital belonged to himself. In particular, he claimed not only the solution of de Beaune's problem and everything else of real interest in l'Hôpital's papers, but also all but three or four pages of the Analyse, which he said was nothing but the first part of the Course on Differential and Integral Calculus that he had given or dictated to l'Hôpital in Paris. Indeed, it was he who had taught the Marquis the new calculus in 1691, giving him instruction for nearly a year. In his published memoirs, Bernoulli was less positive, though his claims increased with time after the death of l'Hôpital. Since Bernoulli was far from reticent in proclaiming his own when others, even his best friends and closest relatives, were involved, the moderation and lateness of his accusations against l'Hôpital naturally caused them to be doubted. Apparently only Leibniz and some Basel friends believed Bernoulli, and in France his claims were regarded as ridiculous. In 1742 Bernoulli published Part II, the Integral Calculus, of his Course of 1601-1602, but not Part I, whose contents, it was noted, had gone into the well known Analyse of l'Hôpital.

Indeed, in the preface there is a famous passage in which l'Hôpital expresses his especial indebtedness to John Bernoulli and asserts that his book will present the discoveries of various persons without further acknowledgment. Nevertheless, in the text there are in fact many specific acknowledgments to half a dozen persons, but not one to Bernoulli.

A century ago the correspondence of l'Hôpital with Leibniz and Huygens was published. Herein may be traced l'Hôpital's own view, or at least the view he wished his great correspondents to entertain, of his progress in calculus and in writing his treatise. Bernoulli's name is not mentioned. After this, few if any historians allowed any credit to Bernoulli's accusations.

In 1922 Schafheitlin found in the Public Library of Basel and published <sup>1</sup> Part I of John Bernoulli's *Course*. It was exactly as its author had described it, and the work of l'Hôpital was at once reduced to the exposition, not the content. But the explanation is more interesting than the fact, and the explanation is to be found only in the letters of the two principals. The existence and contents of these letters has been known to a limited circle for some decades, but the general public will see them for the first time in this volume. Ten were

<sup>1</sup>Verhandlungen der Naturforschenden Gesellschaft in Basel, 1922/23, Band 34. German zig, 1924. published in a thesis by O. J. Rebel in 1934, but these are not the most informative, and in particular the amazing No. 20, from which I will quote below, is not included. A fair idea of the content of the collection has been given by Spiess in an earlier publication.<sup>2</sup> The *Course* and the letters together *fully* substantiate John Bernoulli's claims in all but some minor matters.

The entire relation between l'Hôpital and John Bernoulli is traced in fascinating detail by Spiess in the special preface, pages 123-157. In this review I will give only an oversimplified summary, urging the reader to enjoy for himself Spiess's own words and the following eighty-seven letters. These spread from December 1692, a month after Bernoulli's return to Basel, to a letter from l'Hôpital's widow in 1707.

For Bernoulli's stay in Paris we must rely on his own autobiography, written just before his death, and on a sequence of unpublished letters detailing his recollections to Pierre de Montmort in 1718. After the famous meeting of the two savants in the salon of Malebranche, when Bernoulli dramatically displayed his unpublished secret weapon,<sup>3</sup> the general formula for the radius of curvature of a curve, l'Hôpital immediately engaged Bernoulli to give him four lessons per week. After six months of this, the scene of instruction shifted to l'Hôpital's chateau in the country for three or four more months, and then Bernoulli returned to Basel.

To be brief, in the following letters we find Bernoulli giving l'Hôpital full information on every current topic of research and full answers to every question. Some of these l'Hôpital wrote out and sent to Huygens or Leibniz. In the case of every problem of major interest to which l'Hôpital has had a claim, a lesson or letter from Bernoulli stands in the background.

How did this happen? We must remember that in 1601 John Bernoulli was twenty-four, an unemployed younger son of a modest mercantile family; while a younger brother of a famous mathematician, he had himself published but one important paper. L'Hôpital was a Marquis of thirty, an established savant; young enough for the ambition of learning and perhaps for learning itself, but old enough for assurance and ease in a worldly society; certain of the income of a Marquis, if somewhat improvident in the use of it. While nowadays the difference in social positions seems a trifle, in 1601 it was surely enough to impress even the ebullient self-confidence of John Bernoulli when, freed of worldly cares, he was accepted as an equal and intimate friend in the elegant establishment whose presiding deity was a charming and witty Marquise. On the other side, while l'Hôpital's originality is annulled and his scientific honesty somewhat tarnished by the relation, not only was his curiosity genuine and extraordinary but also from the moment of meeting it was plain that in the face of his young friend's notorious and unconquerable tactlessness, the Marquis would have to put up with a style to which his breeding had hardly accustomed him.

The precise arrangements made while Bernoulli was in France we do not know. Soon after Bernoulli returned to Basel there arose a crisis over de Beaune's problem. Bernoulli had found the solution in the course of his researches on integral calculus and had put it into his *Course* for l'Hôpital as Lesson IX. While Bernoulli was still his guest, l'Hôpital sent Bernoulli's solution to Huygens, who naturally inferred that the sender was the author, the

<sup>2</sup> "Une édition de l'œuvre des mathématiciens Bernoulli," Archives internationales d'histoire des sciences, 1947, I: 356-362. <sup>8</sup>That Huygens, Newton, and Leibniz knew the essence of this formula did not make it the less secret in Paris, where at this time there was no geometer of the first rank. more so since in an earlier letter l'Hôpital had written that he himself had found a solution. At the same time, l'Hôpital published the solution under a pseudonym. A complicated sequence of published and unpublished claims and veiled insults followed. For the plan he had in mind, l'Hôpital could not afford to notice even an open affront. After some mutual explanations and a delay of more than half a year, during which Bernoulli refrained from sending l'Hôpital anything of importance, l'Hôpital on 17 March 1694 (letter No. 20) proposed the most extraordinary agreement in the history of science:

"I will be happy to give you a retainer of 300 pounds, beginning with the first of January of this year . . . I promise shortly to increase this retainer, which I know is very modest, as soon as my affairs are somewhat straightened out . . . I am not so unreasonable as to demand in return all of your time, but I will ask you to give me at intervals some hours of your time to work on what I request and also to communicate to me your discoveries, at the same time asking you not to disclose any of them to others. I ask you even not to send here to Mr. Varignon or to others any copies of the writings you have left with me; if they are published, I will not be at all pleased. Answer me regarding all this . . ."

Bernoulli's response is lost, but the next letter from l'Hôpital indicates that the acceptance was speedy. From this point on, Bernoulli was a giant enchained. Letters 33-44 contain a scolding from l'Hôpital because Bernoulli, after obediently checking, translating into Latin and transmitting to Leipzig l'Hôpital's solution of a minor problem posed by Sauveur, had been unable to restrain himself from adding a note in which he generalized the problem, identified the resulting curve, and gave for the general case his own analysis consisting in one equation, replacing the 27 used by Sauveur to set the special case. L'Hôpital reminded Bernoulli that he was not to publish, but to send all his works to l'Hôpital, who promised to keep them secret, asserting that he had no desire to take for himself the honor of these discoveries (Letter 42). In excusing himself, Bernoulli acknowledged his faults and promised, "You have only to let me know your definite wishes, if I am to publish nothing more in my life, for I will follow them precisely and nothing more by me will be seen." When he wrote those lines in 1695, Bernoulli was as brilliant a mathematician as any living.

As soon as the Analyse appeared, the financial arrangement lapsed.

We should not judge l'Hôpital's procedure too harshly. While perhaps financial necessity compelled Bernoulli to accept the arrangement initially, it continued after he had settled in his professorship at Groningen in 1695. L'Hôpital, being a nobleman, was accustomed to pay for the services of others, and what he did would not then have been considered wrong had Bernoulli been a politician, a lawyer, perhaps even an architect. Certainly it was nothing for l'Hôpital to be proud of. Careful examination of the letters in which l'Hôpital reported his mathematical progress to Leibniz and Huygens shows that with one or two possible exceptions l'Hôpital *did not lie*, but rather referred to Bernoulli in a condescending tone without acknowledging any debt whatever to him and in matters of provenance wrote in such a way as to suggest without actually asserting.

Very soon John Bernoulli realized what he had sold away. The financial returns were ephemeral, and even for the few years the agreement was in force l'Hôpital did not always pay the full sum due. (It would be unfair to suppose Bernoulli was his only disappointed creditor.) In Bernoulli's old age, he boasted of the princely sum for which l'Hôpital had engaged him, magnifying both the amount and the duration.

In the development of calculus as a tool in geometry and mechanics, nearly every letter from John Bernoulli to l'Hôpital is an individual achievement. What is most remarkable is the lightning speed of Bernoulli's conception. His thought and expression in French are no less masterful and far clearer and more direct than in his published works in Latin or his later letters. It would be wasteful to attempt here even to name the problems treated, since these are most easily followed by aid of an index at the end of the volume. I can find no better summary of my impression from these letters than the words l'Hôpital himself wrote to Bernoulli in 1695: "I am very sure that there is scarcely a geometer in the world who can be compared to you."

# The Civilized West Looks at Primitive Africa: 1400–1800 A Study in Ethnocentrism

# By Katherine George\*

To be born into a culture has generally implied being supported by it, being upheld, as it were, on a pedestal, from which one might look down with varying degrees of disinterest or antagonism upon other, alien cultures. Hence, the observer of alien cultures has tended to be prejudiced, in the simple sense that he has preferred his own to all other existent cultures and has viewed the strange as a malformed deviant from the familiar. The ego-flattering naïveté of the Aristotelian division of the world's population into Greeks and barbarians, or freemen by nature and slaves by nature, has formed the usual pattern into which men have fitted their observation of human differences.

The category of barbarism is for Aristotle notably inclusive, and without distinction as to region, people, or custom; but in the writings of most commentators upon human affairs variations in the extent of cultural difference have been associated with variations in the kinds of observational judgments made. The greater the extent of cultural difference, the greater is the amount of antagonism or scorn expressed. Primitive cultures, as described by the civilized observer, have suffered in particular, therefore, from the fashion of disparaging the alien; and the commandment, "Thou shalt not bear false

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