Plate 2 A gunbad from Ghazna in Afghanistan, built at the time of Sultan Masʿūd, the eleventh century patron of the polymath al-Bīrūnī

> Bīrūnī's interests were very wide and deep, and he labored in almost all the branches of science known in his time. He was not ignorant of philosophy and the speculative disciplines, but his bent was strongly toward the study of observable phenomena, in nature and in man … about half his total output is in astronomy, astrology, and related subjects, the exact sciences par excellence of those days. Mathematics in its own right came next, but it was invariably applied mathematics.

3.4 ʿUmar al-Khayyāmī

ʿUmar al-Khayyāmī must be the only famous mathematician to have had clubs formed in his name. They were not, however, clubs to study his many contributions to science, but to read and discuss the famous verses ascribed to him under the title of *The Rubʿāyāt* (*Quatrains*) which have been translated into so many of the world's languages. Indeed, outside of the Islamic world ʿUmar is admired more as a poet than as a mathematician, and yet his contributions to the sciences of mathematics and astronomy were of the first order.

He was born in Nishāpūr, in a region now part of Iran, but then known as Khurasān, around the year 1048. This is only shortly before the death of al-Bīrūnī, at a time when the Seljuk Turks were masters of Khurasān, a vast region east of what was then Iran, whose principal cities were Nishapur, Balkh, Marw and Tūs. His name, "al-Khayyāmī," suggests that either he or his father at one time practiced the trade of tentmaking (al-khayyām = tentmaker). In addition, he showed an early interest in the mathematical sciences by writing treatises on arithmetic, algebra and music theory, but, beyond these facts, nothing is known of his youth. The nice story of a boyhood pact with a schoolmate, who later was known as Niẓām al-Mulk and became a minister in the government of the ruler Malikshah, to the effect that whichever of them first obtained high rank would help the other is not supported by the dates at which these men lived. In fact, most scholars believe that ʿUmar died around 1131, so if he had been Niẓām's schoolmate, he would have had to be around 120 years old when he died in order for the story to fit the known dates for Niẓām al-Mulk. Better founded is the report of the biographer Ẓāhir al-Dīn al-Bayhaqī. He knew al-Khayyāmī personally and describes him as being both ill-tempered and narrow-minded. Of course, al-Khayyāmī had examined al-Bayhaqī as a schoolboy in literature and mathematics, so it may be that he did not get to know him under the best of circumstances.

We also know that in 1070, when ʿUmar wrote his great work on algebra, he was supported by the chief judge of Samarqand, Abū Ṭāhir. In this work, ʿUmar systematically studied all the kinds of cubic equations and used conic sections to construct the roots of these equations as line segments obtained from the intersections of these curves. There is evidence that ʿUmar also tried to find an algebraic formula for these roots, for he wrote that "We have tried to express these roots by algebra but have failed. It may be, however, that men who come after us will succeed." The candor of this passage, and its recognition of being part of a tradition of inquiry that will continue after one's own death, bespeaks, al-Bayhaqī aside, a modest and civilized man.

During the 1070s, ʿUmar went to Isfahan (see Plate 3), where he stayed for 18 years and, with the support of the ruler Malikshah and his minister Niẓām al-Mulk, conducted a program of astronomical investigations at an observatory. As a result of these researches, he was able, in 1079, to present a plan to reform the calendar then in use. (One wonders if there is not an echo of this achievement, together with a nice reference to squaring the circle, in the quatrain "Ah, but my calculations, people say,/Have squared the year to human compass, eh?/If so by striking out/Unborn tomorrow and dead yesterday.") ʿUmar's scheme made eight of every 33 years leap years, with 366 days each, and produced a length for the year closer to the true value than does the present-day Gregorian calendar.

Another important work of ʿUmar's was his *Explanation of the Difficulties in the Postulates of Euclid*, a work composed in 1077, two years before he presented his calendar reform. In this treatise, ʿUmar treats two extremely important questions in

Plate 3 The entrance to the Friday Mosque in Isfahan, Friday being the day when Muslims gather together in the mosque to pray and, perhaps, to hear a khutba (homily). Parts of the mosque date back to the time of ʿUmar al-Khayyāmī. The tile designs on the twin minarets are calligraphic renderings of the word Allāh (The One God), and calligraphy borders the geometric patterns of arabesques on the facade

the foundations of geometry. One of these, already treated by Thābit ibn Qurra and Ibn al-Haytham (known to the West as Alhazen), is the fifth postulate of Book I of *Euclid's Elements* on parallel lines. (In fact, Toth provides evidence that remarks in

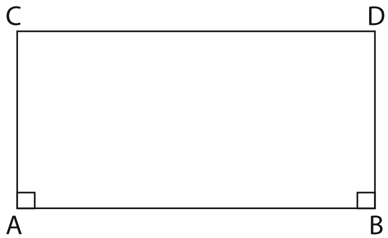

Fig. 3 CA and DB are equal straight lines perpendicular to a given straight line, AB, at points A and B respectively. CD is a straight line joining C and D. It is required to show,without the use of the parallel postulate, that the angles at C and D are right

various writings of Aristotle imply that mathematicians before Euclid investigated this question.) ʿUmar bases his analysis on the quadrilateral ABCD of Fig. 3, where CA and DB are two equal line segments, both perpendicular to AB, and he recognizes that in order to show that the parallel postulate follows from the other Euclidean postulates it suffices to show that the interior angles at C and D are both right angles, which implies the existence of a rectangle. (In fact, the two may be shown to be equivalent.) Although Ibn al-Haytham, who flourished around 1010, preceded ʿUmar in using this method of attack on the problem, ʿUmar took issue with Ibn al-Haytham's use of motion in geometry. A century and a half later Naṣīr al-Dīn al-Ṭūsī adopted ʿUmar's quadrilateral when he wrote his treatment of Euclid's parallel postulate. Then, one of his followers, usually referred to as "Pseudo-Ṭūsī," wrote an exposition of *Euclid's elements*. This work, published in Arabic in Rome in 1594 under al-Ṭūsī's name, was translated into Latin and influenced both J. Wallis and G. Saccheri's work on the postulate in the seventeenth and eighteenth centuries.

The other topic that ʿUmar treated in his discussion of the difficulties in Euclid is that of ratios. Here, al-Khayyāmī's achievements are twofold. The one is his demonstration that a definition of proportion that was elaborated in Islamic mathematics, a definition that he felt was more true to the intuitive idea of "ratio," was equivalent to the definition Euclid used. The other is his suggestion that the idea of number needed to be enlarged to include a new kind of number, namely ratios of magnitudes. For example, in ʿUmar's view, the ratio of the diagonal of a square to the side $(\sqrt{2})$, or the ratio of the circumference of a circle to its diameter (π), should be considered as new kinds of numbers. This important idea in mathematics amounted to the introduction of positive real numbers and, as was the case with the parallel postulate, this was communicated to European mathematicians through the writings of the pseudo-Naṣīr al-Dīn al-Ṭūsī.

ʿUmar once said to a friend that when he died he wanted to be buried in Isfahan, where "the wind will blow the scent of the roses over my grave." His wish was granted and the tomb of Islam's poet–mathematician has remained there to this day.