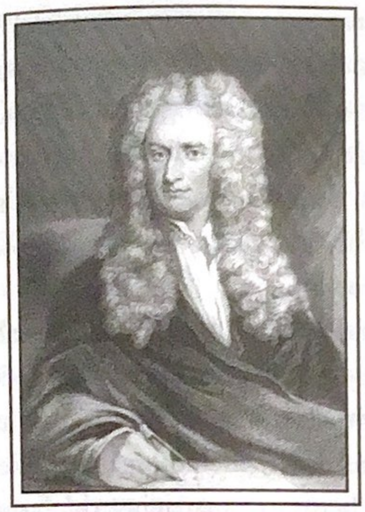


Isaac Newton

(1642–1727)

HIS LIFE AND WORK

Galileo died on January 8, 1642, exactly three hundred years before the day I was born. Isaac Newton was born on Christmas Day of that year in the English industrial town of Woolsthorpe, Lincolnshire. He would later become Lucasian Professor of Mathematics at Cambridge University, the chair I now hold.

Newton's mother did not expect him to live long, as he was born very prematurely; he would later describe himself as having been so small at birth he could fit into a quart pot. Newton's yeoman father, also named Isaac, had died three months earlier, and when Newton reached two years of age, his mother, Hannah Ayscough, remarried, wedding Barnabas Smith, a rich clergyman from North Witham. Apparently there was no place in the new Smith family for the young Newton, and he was placed in the care of his grandmother, Margery Ayscough. The specter of this abandonment, coupled with the tragedy of never having known his father, haunted Newton for the rest of his life. He despised his stepfather; in journal entries for 1662, Newton recalled "threatening my father and mother Smith to burne them and the house over them."

Much like his adulthood, Newton's childhood was filled with episodes of harsh, vindictive attacks, not only against perceived enemies but also against friends and family. He also displayed the kind of curiosity early on that would define his life's achievements, taking an interest in mechanical models and architectural drawing. Newton spent countless hours building clocks, flaming kites, sundials, and miniature mills (powered by mice), as well as drawing elaborate sketches of animals and ships. At the age of five, he attended schools at Skillington and Stoke but was considered one of the poorest students, with his teachers' reports stating he was "inattentive" and "idle." Despite his curiosity and demonstrable passion for learning, he was unable to apply himself to schoolwork.

By the time Newton reached the age of ten, Barnabas Smith had passed away and Hannah had come into a considerable sum from Smith's estate. Isaac and his grandmother began living with Hannah, a half-brother, and two half-sisters. Because his work at school was uninspiring, including his studies of mathematics, Hannah decided that Isaac would be better off managing the farm and estate, and she pulled him out of the Free Grammar School in Grantham. Unfortunately for her, Newton had even less skill or interest in managing the family estate than he had in schoolwork. Hannah's brother William, a clergyman, decided that it would be best for the family if the absent-minded Isaac returned to school to finish his education.

This time, Newton lived with the headmaster of the Free Grammar School, John Stokes, and he seemed to turn a corner in his education. One story has it that a blow to the head, administered by a schoolyard bully, somehow enlightened him, enabling the young Newton to reverse the negative course of his educational promise. Now demonstrating intellectual aptitude and curiosity, Newton began preparing for further study at a university. He decided to attend Trinity College, his uncle William's alma mater, at Cambridge University.

At Trinity, Newton became a subsizar, receiving an allowance toward the cost of his education in exchange for performing various chores such as waiting tables and cleaning rooms for the faculty. But by 1664, he was elected scholar, which status guaranteed him financial support and freed him from menial duties. When the university closed because of the bubonic plague in 1665, Newton retreated to Lincolnshire. In the eighteen months he spent at home during the plague, he devoted himself to mechanics and mathematics and began to concentrate on optics and gravitation. This "annus mirabilis" (miraculous year), as Newton called it, was one of the most productive and fruitful periods of his life. It is also around this time that an apple, according to legend, fell onto Newton's head, awakening him from a nap under a tree and spurring him on to define the laws of gravity. However far-fetched the tale, Newton himself wrote that a falling apple had "occasioned" his foray into gravitational contemplation, and he is believed to have performed his pendulum experiments then. "I was in the prime of my age for invention," Newton later recalled, "and minded Mathematicks and Philosophy more than at any time since."

When he returned to Cambridge, Newton studied the philosophy of Aristotle nd Descartes, as well as the science of Thomas Hobbes and Robert Boyle. He was aken by the mechanics of Copernicus and Galileo's astronomy, in addition to Kepler's ptics. We have little direct information on Newton's mathematical education prior o his matriculation at Cambridge. Newton's first tutor at Cambridge was Benjamin ulleyn, who later became Regius Professor of Greek. Newton soon came under the utelage of Isaac Barrow, an outstanding mathematician and one of the founders of he Royal Society, who tutored Newton as young Isaac sped through Euclid's *Elements*. Following this, Newton soon mastered works on algebra by William Oughtred 1574–1660) and François Viète (1540–1603) and, most importantly, Descartes' *Geometrie*.

Around this time, Newton began his prism experiments in light refraction and dispersion, possibly in his room at Trinity or at home in Woolsthorpe. A development at the university that clearly had a profound influence on Newton's future—was the arrival of Isaac Barrow, who had been named the Lucasian Professor of Mathematics. Barrow recognized Newton's extraordinary mathematical talents, and when he resigned his professorship in 1669 to pursue theology, he recommended the twenty-seven-year-old Newton as his replacement.

Newton's first studies as Lucasian Professor were centered in the field of optics. He set out to prove that white light was composed of a mixture of various types of light, each producing a different color of the spectrum when refracted by a prism. His series of elaborate and precise experiments to prove that light was composed of minute particles drew the ire of scientists such as Hooke, who contended that light traveled in waves. Hooke challenged Newton to offer further proof of his eccentric optical theories. Newton's way of responding was one he did not outgrow as he matured. He withdrew, set out to humiliate Hooke at every opportunity, and refused to publish his book, *Opticks,* until after Hooke's death in 1703.

Early in his tenure as Lucasian Professor, Newton was well along in his study of pure mathematics, but he shared his work with few of his colleagues. Already by 1666, he had discovered general methods of solving problems of curvature—what he termed "theories of fluxions and inverse fluxions." The discovery set off a dramatic feud with supporters of the German mathematician and philosopher Gottfried Wilhelm Leibniz, who more than a decade later published his findings on differential and integral calculus. Both men arrived at roughly the same mathematical principles, but Leibniz published his work before Newton. Newton's supporters claimed that Leibniz had seen the Lucasian Professor's papers years before, and a heated argument between the two camps, known as the Calculus Priority Dispute, did not end until Leibniz died in 1716. Newton's vicious attacks, which often spilled over to touch on views about God and the universe, as well as his accusations of plagiarism, left Leibniz impoverished and disgraced.

Most historians of science believe that the two men in fact arrived at their ideas independently and that the dispute was pointless. Newton's vitriolic aggression toward Leibniz took a physical and emotional toll on Newton as well. He soon found himself involved in another battle, this time over his theory of color and with the English Jesuits, and in 1678 he suffered a severe mental breakdown. The next year, his mother passed away, and Newton began to distance himself from others. In secret he delved into alchemy, a field widely regarded already in Newton's time as fruitless. This episode in the scientist's life has been a source of embarrassment to many Newton scholars. Only long after Newton died did it become apparent that his interest in chemical experiments was related to his later research in celestial mechanics and gravitation.

Newton had already begun forming theories about motion by 1666, but he was as yet unable to adequately explain the mechanics of circular motion. Some fifty years earlier, the German mathematician and astronomer Johannes Kepler had proposed three laws of planetary motion, which accurately described how the planets moved in relation to the sun, but he could not explain why the planets moved as they did. The closest Kepler came to understanding the forces involved was to say that the sun and the planets were "magnetically" related.

Newton set out to discover the cause of the planets' elliptical orbits. By applying his own law of centripetal force to Kepler's third law of planetary motion, (the law of harmonies) he deduced the inverse-square law, which states that the force of gravity between any two objects is inversely proportional to the square of the distance between the object's centers. Newton was thereby coming to recognize that gravitation is universal—that one and the same force causes an apple to fall to the ground and the moon to race around the earth. He then set out to test the inverse-square relation against known data. He accepted Galileo's estimate that the moon's distance from the earth is sixty earth radii, but the inaccuracy of his own estimate of the earth's diameter made it impossible to complete the test to his satisfaction. Ironically, it was an exchange of letters in 1679 with his old adversary Hooke that renewed his interest in the problem. This time, he turned his attention to Kepler's second law, the law of equal areas, which Newton was able to prove held true because of centripetal force. Hooke, too, was attempting to explain the planetary orbits, and some of his letters on that account were of particular interest to Newton.

At an infamous gathering in 1684, three members of the Royal Society—Robert Hooke, Edmond Halley, and Christopher Wren, the noted architect of St. Paul's

Cathedral—engaged in a heated discussion about the inverse-square relation governing the motions of the planets. In the early 1670s, the talk in the coffeehouses of London and other intellectual centers was that gravity emanated from the sun in all directions and fell off at a rate inverse to the square of the distance, thus becoming more and more diluted over the surface of the sphere as that surface expands. The 1684 meeting was, in effect, the birth of *Principia*. Hooke declared that he had derived from Kepler's law of ellipses the proof that gravity was an emanating force, but would withhold it from Halley and Wren until he was ready to make it public. Furious, Halley went to Cambridge, told Newton Hooke's claim, and proposed the following problem. "What would be the form of a planet's orbit about the sun if it were drawn towards the sun by a force that varied inversely as the square of the distance?" Newton's response was staggering. "It would be an ellipse," he answered immediately, and then told Halley that he had solved the problem four years earlier but had misplaced the proof in his office.

At Halley's request, Newton spent three months reconstituting and improving the proof. Then, in a burst of energy sustained for eighteen months, during which he was so caught up in his work that he often forgot to eat, Newton further developed these ideas until their presentation filled three volumes. He chose to title the work *Philosophiae naturalis principia mathematica,* in deliberate contrast with Descartes' *Principia philosophiae.* The three books of Newton's *Principia* provided the link between Kepler's laws and the physical world. Halley reacted with "joy and amazement" to Newton's discoveries. To Halley, it seemed the Lucasian Professor had succeeded where all others had failed and he personally financed publication of the massive work as a masterpiece and a gift to humanity.

Where Galileo had shown that objects were "pulled" toward the center of the earth, Newton was able to prove that this same force, gravity, affected the orbits of the planets. He was also familiar with Galileo's work on the motion of projectiles, and he asserted that the moon's orbit around the earth adhered to the same principles. Newton demonstrated that gravity explained and predicted the moon's motions as well as the rising and falling of the tides on earth. Book 1 of *Principia* encompasses Newton's three laws of motion:

1. Every body perseveres in its state of resting, or uniformly moving in a right line, unless it is compelled to change that state by forces impressed upon it.

2. The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

3. To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary directions.

Book 2 began for Newton as something of an afterthought to Book 1; it was not included in the original outline of the work. It is essentially a treatise on fluid mechanics, and it allowed Newton room to display his mathematical ingenuity. Toward the end of the book, Newton concludes that the vortices invoked by Descartes to explain the motions of planets do not hold up to scrutiny, for the motions could be performed in free space without vortices. How that is so, Newton wrote, "may be understood by the first Book; and I shall now more fully treat of it in the following Book."

In Book 3, subtitled *System of the World,* by applying the laws of motion from Book 1 to the physical world Newton concluded, "there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain." He thus demonstrated that his law of universal gravitation could explain the motions of the six known planets, as well as moons, comets, equinoxes, and tides. The law states that all matter is mutually attracted with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Newton, by a single set of laws, had united the earth with all that could be seen in the skies. In the first two "Rules of Reasoning" from Book 3, Newton wrote:

> *We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. Therefore, to the same natural effects we must, as far as possible, assign the same causes.*

It is the second rule that actually unifies heaven and earth. An Aristotelian would have asserted that heavenly motions and terrestrial motions are manifestly not the same natural effects and that Newton's second rule could not, therefore, be applied. Newton saw things otherwise. *Principia* was moderately praised on its publication in 1687, but only about five hundred copies of the first edition were printed. However, Newton's nemesis, Robert Hooke, had threatened to spoil any coronation Newton might have enjoyed. After Book 2 appeared, Hooke publicly claimed that the letters he had written in 1679 had provided scientific ideas that were vital to Newton's discoveries. His claims, though not without merit, were abhorrent to Newton, who vowed to delay or even abandon publication of Book 3. Newton ultimately relented and published the final book of *Principia,* but not before painstakingly removing from it every mention of Hooke's name.

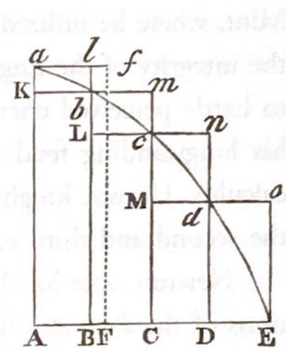
Newton's work on integral and differential calculus can be found in his notebook from the mid-1660s. However, Newton never published a purely mathematical text of his own. Only in the second half of the twentieth century have an extensive portion of his mathematical papers been published. Newton gave his contemporaries a glimpse of his discoveries in the calculus in Book I, Section 1, of the *Principia.* He titled th section "The method of first and last ratios of quantities, by the help of which we demonstrate the propositions that follow." In this section, included in the present volume, Newton presents eleven "lemmas," a Greek term for "subsidiary proposition" on first and last ratios that will allow him to make interchangeable use of figures constructed with curves and corresponding figures constructed from straight lines.

In the first lemma Newton proves that

> *Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer to each other than by any given difference, become ultimately equal.*

He proves this in a very straightforward manner that anticipates Weierstrass's epsilon-delta method two centuries later. If the quantities and the ratios do not ultimately become equal, then they will have some finite ultimate difference *D* and they cannot ultimately approach each closer than that difference *D*! Notice that Newton frames this statement in terms of change over time. Given the setting in the *Principia*, a work on physical science, this is hardly surprising.

Two millennia before Newton, Archimedes had proven theorems about the area of particular geometric objects such as the circle by inscribing and circumscribing polygons about the object. In the second lemma, Newton takes a cue from Archimedes and extends this method to an arbitrary curve by inscribing and circumscribing rectangles about portions of the curve and demonstrating that the inscribed and circumscribed figures composed of these inscribed and circumscribed rectangles have areas that have an ultimate ratio of equality.



Newton has the reader consider an arbitrary curve in relation to a straight line represented here by AE that he divides into equal parts AB, BC, CD, and so forth, that will be diminished *ad infinitum.* He then constructs the rectangles such as AKbB inscribed within a segment of the curve and AalB circumscribed about the same segment of the curve. He notes that the difference in the areas of these rectangles is the area of the rectangle aKbl and that the sum of the areas of these "difference" rectangles is simply the area of the first circumscribed rectangle AalB! And then Newton clinches the proof noting that the length of the base AB of the rectangle AalB is being diminished *ad infinitum* so that the area of the rectangle AalB "becomes less than any given space." Hence, the areas of the inscribed and circumscribed figures ultimately "become ultimately equal one to the other" and the area described by the curvilinear figure as well!

Newton put his lemmas to immediate use in the first proposition he demonstrates in the *Principia:* Kepler's Area Law!

In a Scholium at the end of Section 1, Newton addresses concerns that would soon be voiced by critics that quantities that themselves vanish to zero cannot have an ultimate proportion as they vanish to zero. He responds by comparing ultimate ratios to a body's velocity at a particular point in space. In an argument having its roots in the ancient Greeks, Newton points out that the body is certainly not at rest for the moment that it is at the particular place. In contrast, it has a definite ultimate velocity, for the moment that it is at the particular place, just as quantities that vanish to zero can have an ultimate ratio to each other as the vanish to zero.

> *For those ultimate ratios with which quantities vanish are not truly the ratios of ultimate quantities, but limits towards which they approach nearer than by any given difference, but never go beyond, nor in effect attain to, till the quantities are diminished* in infinitum.

Newton began the eighteenth century in a government post as warden of the Royal Mint, where he utilized his work in alchemy to determine methods for reestablishing the integrity of the English currency. As president of the Royal Society, he continued to battle perceived enemies with inexorable determination, in particular carrying on his longstanding feud with Leibniz over their competing claims to have invented calculus. He was knighted by Queen Anne in 1705, and lived to see publication of the second and third editions of the *Principia.*

Newton occasionally claimed that he had derived many of the major propositions of the *Principia* using his discoveries in the calculus. He made one such claim in an unpublished preface to the *Principia* that he drafted around 1715 and finally went into print with such a claim in an anonymously published 1722 review of a book assessing his and Leibniz's claims to have invented calculus. He wrote:

> *By the help of the new analysis Mr. Newton found out most of the propositions of his* Principia Philosophiae: *but because the ancients for making things certain admitted nothing into geometry before it was demonstrated synthetically, he demonstrated the propositions synthetically, that the system of the heavens might be founded upon good geometry. And this makes it difficult now for unskillful men to see the analysis by which the propositions were found out.*

Recent scholarly analysis of Newton's notebooks have revealed that there is not a shred of evidence for his extravagant claims made to secure priority for discovering the calculus for himself and not for his arch-rival Leibniz.

Isaac Newton died in March 1727, after bouts of pulmonary inflammation and gout. As was his wish, Newton had no rival in the field of science. The man who apparently formed no romantic attachments with women (some historians have speculated on possible relationships with men, such as the Swiss natural philosopher Nicolas Fatio de Duillier) cannot, however, be accused of a lack of passion for his work. The poet Alexander Pope, a contemporary of Newton's, most elegantly described the great thinker's gift to humanity:

Nature and Nature's laws lay hid in night:
God said, "Let Newton be! and all was light."

For all the petty arguments and undeniable arrogance that marked his life, toward its end, Isaac Newton was remarkably poignant in assessing his accomplishments: "I do not know how I may appear to the world, but to myself I seem to have been only like a boy, playing on the sea-shore, and diverting myself, in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."