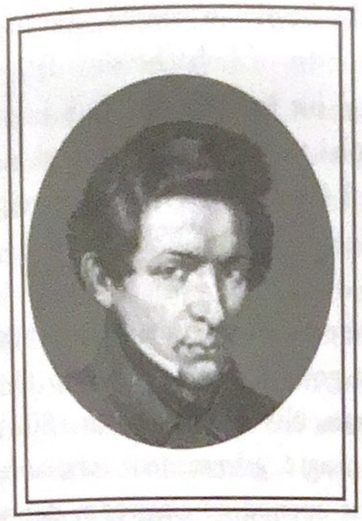


Nikolai Ivanovich Lobachevsky

(1792–1856)

HIS LIFE AND WORK

In the Sherlock Holmes story *The Sign of Four,* Arthur Conan Doyle has the great detective make the following pronouncement.

> When you have eliminated the impossible, whatever remains, *however improbable,* must be the truth.

hus it was with geometry at the dawn of the nineteenth century. After two mil-nnia of trying to prove Euclid's parallel postulate, it was time for geometers to emonstrate that it could not be proved and that other geometries were possible, owever improbable that might be. This is the nearly concurrent accomplishment of e Russian mathematician Nikolai Ivanovich Lobachevsky and the Hungarian math-matician János Bolyai presented in this and the next chapter respectively.

Nikolai Ivanovich Lobachevsky was born on December 1, 1792 in the little town f Makariev, about 40 miles north of the city of Nizhny Novgorod on the Volga River the heart of Russia. He was the second of three sons born to Ivan Maximovich and

Praskovia Alexandrovna Lobachevsky, a couple who had emigrated from Poland a few years earlier. Little is known of Lobachevsky's father except that he was barely able to support his family on his income as a clerk in a government land surveying office. Ivan died when Nikolai was seven. Hoping to make a living teaching young children, the newly widowed Praskovia moved her family 200 miles east to the city of Kazan, almost on the edge of Siberia. Nikolai would spend the rest of his life in this new hometown.

Praskovia's three sons were her first students and they turned out to be prize students. Each of them won a government scholarship to pay for their attendance at the local gymnasium in Kazan. Nikolai entered in 1802 and graduated with highest honors five years later, winning a government scholarship to attend the local university. Kazan University first opened its classroom doors in 1805, the result of one of the many educational reforms that marked the early years of the reign of Tsar Alexander I.

The very newness of the university coupled with its remoteness meant that it did not have an entrenched faculty. Instead, it hired young faculty from all over eastern and central Europe. Indeed, when Lobachevsky was a student, Kazan University was able to boast of more accomplished scholars than all but the very best German universities. These young scholars brought the latest ideas in mathematics, physics, astronomy, philosophy, and pedagogy to Kazan.

The most important of these for Lobachevsky was Martin Bartels (1769–1833) who had only been able to find employment in German teaching mathematics in a school. However, Bartels had had the good fortune to have had the great Gauss as one of his students, and they continued to correspond with each other over the years.

Bartels' lectures soon persuaded Lobachevsky to abandon his plans to study medicine. Bartels lectured on Euler's treatment of differential and integral calculus, Lagrange's analytical mechanics, Gauss's *Disquisitiones arithmeticae,* Monge's work on the application of mathematical analysis to geometry, and Montucla's four volume history of mathematics. Montucla's detailed coverage of the history of attempts to prove Euclid's parallel postulate may well have been the source of Lobachevsky's interest in the problem.

In many ways Lobachevsky did not display model behavior as a student. From time to time, university administrators accused him of being "obstinate", "overly self-centered", and having atheistic tendencies. In August 1811, the curator of the Kazan school district recommended to the University Council that it should express to Lobachevsky, "the leader in bad conduct, our regrets about his unbecoming actions which becloud his superior abilities." Fortunately for Nikolai, Bartels and his other professors knew better than the administrators. They recommended that he be granted a master's degree in recognition of his "extraordinary gifts in mathematical and physical sciences," which he received at the end of the year.

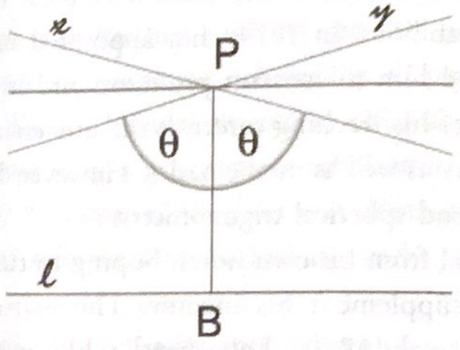
In those days, it was no easy matter finding a teaching job in Kazan. Nikolai must have considered it a stroke of good fortune when his elder brother Alexis took sick leave and was unable to continue teaching courses in elementary mathematics to government employees in need of a college degree to secure promotion to a higher position. The authorities at the university must have been impressed by the former troublemaker's teaching abilities. In 1814, they appointed him to a lectureship, two years later they promoted him to assistant professor, and in 1822, to full professor. Throughout the course of his teaching career at the university, Lobachevsky lectured on physics and astronomy as well as mathematics. However, he always felt most comfortable teaching plane and spherical trigonometry.

Lobachevsky lectured from his own notes, hoping to turn them into a textbook that would eventually supplement his income. The manuscripts for these notes reveal that up until the early 1820s, Lobachevsky, like many before him, tried to prove Euclid's parallel postulate. These manuscripts included three attempted proofs of the postulate, but elsewhere in the notes, Lobachevsky admitted that none of them were conclusive.

We cannot be sure what motivated Lobachevsky to discard his attempts to prove the parallel postulate. His colleague, Bartels, may have passed on hints received from the great Gauss. Alternatively, he may have been inspired by the philosopher A. S. Lubkin, who took up his post at Kazan in 1812 and taught a strict form of empiricism that rejected Immanuel Kant's treatment of space as a category that is synthetic and *a priori* that organizes our experience rather than being an experience derived from them. Lobachevsky would later write:

> *The fruitlessness of the attempts made, since Euclid's time, for the space of 2000 years, aroused in me the suspicion that the truth, which it was desired to prove, was not contained in the data themselves; that to establish it the aid of experiment would be needed, for example, of astronomical observations, as in the case of the laws of nature.*

Here is a brief recapitulation of Lobachevsky's 1840 essay *Geometrical Researches on the Theory of Parallels*, his most mature exposition and the work reproduced here. He begins by considering a point P, and a straight line l in the same plane as P but not containing P. He lets PB be the line through P perpendicular to l. He next considers the line through P perpendicular to PB. In Euclidean geometry this is the only line through P that does not intersect the line l. That is precisely the assumption that Lobachevsky abandons in his construction of a non-Euclidean geometry when he *postulates* that there are other lines through P that do not intersect the line l. These non-intersecting lines are separated from the lines through P that do intersect the line l by the two lines x and y shown in the figure below. The lines x and y do not do intersect the line l either. Lobachevsky defines *them* as the parallels to the line l. Each is said to have a definite *direction of parallelism:* x is the parallel to the right and y is the parallel to the left.



The angle θ that the perpendicular PB makes with each of the two parallels is called the *angle of parallelism* for the length PB. In ordinary Euclidean geometry $\theta = 90°$. In Lobachevsky's geometry θ is a function of the distance from P to B. It approaches 0° as the distance becomes infinitely larger and approaches 90° as the distance becomes infinitesimally small.

Having made this alternate postulate, Lobachevsky goes on to prove that in this alternate geometry.

- If line l_1 is parallel to line l_2 for a given point on l_1, then l_1 is parallel to l_2 for *every* point of l_1. (The property called "Permanency.")
- If line l_1 is parallel to line l_2, then line l_2 is parallel to line l_1. ("Reciprocity")
- If line l_1 is parallel to line l_0 and if line l_2 is parallel to line l_0 in the same direction, then line l_1 is parallel to line l_2. ("Transitivity")

So far, none of these conclusions is startling. Each one has a direct analog in Euclidean geometry. However, the next conclusion is quite startling having no analog in Euclidean geometry.

- If line l_1 is parallel to line l_2, then l_1 is *asymptotic* to l_2, that is l_1 and l_2 come arbitrarily close to each other.

In contrast, in Euclidean geometry, two parallel lines remain equidistant from each other.

Lobachevsky completed his first treatment of non-Euclidean geometry in 1823 in a manuscript he title *Geometriya.* (It was not published until 1909.) In this manuscript, Lobachevsky often used the phrase "imaginary geometry" for the new geometry he had discovered. The manuscript makes it clear that he considered Euclidean geometry to be a special case of this more general "imaginary geometry" just as the real numbers are a special case of the imaginary, i.e. complex numbers.

For some reason, Lobachevsky chose not to communicate his results for over two years when, in February 1826, he read a paper (that no longer survives) titled *A concise outline of the foundations of geometry* to his colleagues in Kazan. Encouraged by his colleagues' reception, Lobachevsky incorporated his results in his lecture series *Elements of Geometry* being published in installments in the *Kazan Messenger* during the academic year 1829–30.

The first quarter of the nineteenth century saw the first significant contribution made by Russian mathematicians. Under the Francophone influence of the Russian court, most mathematicians based in St. Petersburg and Moscow published in French. A few published in German. In far off Kazan, Lobachevsky did not share this view. He submitted his *Elements of Geometry* to the St. Petersburg Academy of Science for republication in one of their scholarly journals. The referee Mikhail Ostrogradski, a vocal proponent of the view that that complex mathematical arguments simply could not be presented in Russian, judged the work to be unworthy of publication by the Academy.

However, someone in St. Petersburg, most likely Ostrogradski, paid some attention to Lobachevsky. Shortly after the rejection, the St. Petersburg journal *Son of the Fatherland* carried a scathing unsigned review of the *Elements of Geometry.* The review severely criticized Lobachevsky for nor making a sufficient effort to present his ideas clearly. Certainly this criticism would have been no surprise coming from someone who thought Russian too crude a language for mathematics.

Lobachevsky might have fared better with the initial reception of his work had Kazan University been able to retain its cosmopolitan faculty when they could have most come to his assistance. Unfortunately, in 1826, the government of the new Tsar Nicholas I forced the University to dismiss many of its foreign faculty including, especially, Bartels. When Lobachevsky revised his work in *New Elements of Geometry* he chose not to communicate it to the Academy in St. Petersburg. Instead, he merely published it in the *Scientific Journal of Kazan University* between 1835 and 1838.

Bypassing the Academy in St. Petersburg, Lobachevsky also submitted a short paper written in French titled *Géométrie imaginaire* providing an overview of his work to the *Journal of Pure and Applied Mathematics.* This journal, published by August Crelle in Berlin, had quickly established itself as Europe's leading mathematics journal since its founding in 1826. This paper undoubtedly reached a wide audience, but only the great Gauss seemed to notice it. The astronomer Otto Wilhelm Struve reports that when he visited Gauss in 1843 he found the great man reading one of

Lobachevsky's works in its original Russian, a language Gauss did not conquer until he was well into his sixties. At Gauss's suggestion perhaps, Lobachevsky published a German version of his work in 1840 in the essay *Geometrical Researches on the Theory of Parallels.*

In addition to his great work in geometry, Lobachevsky also performed great works on behalf of his university. He was put in charge of the library in 1825 and made rector in 1827, a post he held for nineteen years. In this position, he supervised the expansion of the university's scientific teaching, both in terms of buildings and curriculum. In 1828, he delivered a famous speech on the role of science in education. Influenced by the spirit of the eighteenth century Enlightenment, he argued that education should be the primary path to "honor and glory" in Russian society in contrast to nobility of blood, which dominated much of Russia.

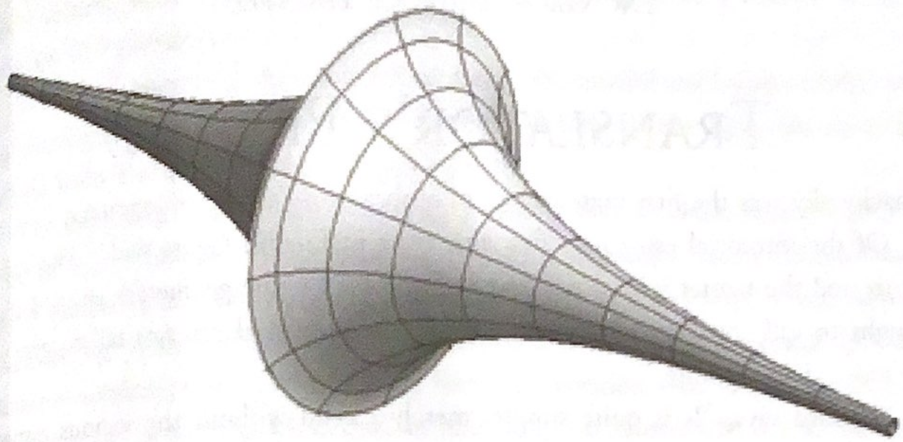
Lobachevsky's concern for his university often manifested itself in surprising fashions. When a cholera epidemic erupted in Kazan in 1830, Lobachevsky invited faculty, students, and their families to move onto the university campus. He closed the gates to everyone except doctors and enforced strict sanitary regulations. This helped save many in the university community. In 1842, a disastrous fire swept through the city, destroying half of it. The university lost many of its buildings, including the recently completed library. Fortunately, Lobachevsky had the presence of mind to save the instruments and the books housed in the library. In two years they had a new home thanks to his efforts.

In 1837, Tsar Nicholas I made Lobachevsky a Boyar, a member of the hereditary nobility, in recognition of his many contributions to higher education. This recognition was mostly forgotten nearly a decade later when, in 1846 as he was suffering from arterioscelrosis, a new local administration abruptly stripped Lobachevsky of his professorship and rectorship thinking him too old and sick at age 53. Lobachevsky retired to family life on his estate.

Lobachevsky did not marry until he was nearly 40. When he did in 1832, he married the Lady Varvara Alexievna Moiseiva, a much younger woman who came from a wealthy family from Kazan. Together they had seven children. Their oldest son became Nikolai's pride and joy. Tragically, this son died shortly after his father's forced retirement, putting Nikolai into a deep depression. Soon afterwards, Lobachevsky's illness worsened and he lost his eyesight. He died on February 24, 1856.

Lobachevsky died having made a significant impression on only a few of his contemporaries. Gauss was one of those on whom he did make an impression. In 1842, Gauss persuaded the Royal Society of Sciences in Göttingen to make Lobachevsky a corresponding member. At about the same time he was also made an honorary professor by Moscow University.

Further recognition outside of Kazan would have to wait until after Lobachevsky's death. This came first when the French mathematician Jules Hoüel published a French translation of the *Theory of Parallels* in 1865. Three years later, the Italian mathematician Eugenio Beltrami demonstrated that Lobachevsky's geometry could be realized as the surface of a pseudo-sphere in Euclidean space.



Beltrami demonstrated that in Lobachevsky's geometry, the sum of the angles of a triangle is less than two right angles. Most importantly, Beltrami's techniques demonstrated that a contradiction in Lobachevsky's geometry would manifest itself as a contradiction in Euclidean geometry.

By 1870, the eminent German mathematician Karl Weierstrass offered a seminar in Lobachevsky's geometry at the prestigious University of Berlin. A few years later the English geometer William Kingdom Clifford described Lobachevsky as "the Copernicus of Geometry." What more fitting recognition could there be?