MA 361 Homework 4

Due Friday, September 25

1. Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$.

2. For any group element a and positive integer k, show that $C(a) \subseteq C(a^k)$. Use this fact to complete the following statement: "In a group, if k is an integer and x commutes with a, then …" Is the converse true?

3. If H is a subgroup of G, then the *centralizer* of H is the set

$$C(H) = \{x \in G | xh = hx \text{ for all } h \in H\}.$$

   Prove that $C(H)$ is a subgroup of G.

4. Prove or disprove: the center of a group is abelian.

5. Prove or disprove: the centralizer of an element of a group is abelian.

6. Find an example of a noncyclic group, all of whose proper subgroups are cyclic.

7. How many subgroups does $\mathbb{Z}_{20}$ have? List a generator for each of these subgroups.

8. Prove that a group of order 1,2, or 3 must be cyclic. Find an example of a group of order 4 that is not cyclic.