MA 565 Homework 2
Due Friday, September 11

Axler Chapter 3A, # 8,10 Axler Chapter 3B, # 2,11,30

1. Let V be a vector space over a field k. Show that V is isomorphic to $\mathrm{Hom}(k, V)$.

2. (a) Give an example of a vector space V and a linear map $T : V \to V$ such that T is injective but not surjective.

   (b) Give an example of a vector space V and a linear map $T : V \to V$ such that T is surjective but not injective.

3. (a) Let V be a vector space. Show that there exists a unique linear map from the trivial vector space to V.

   (b) Let Z be a vector space such that, for any vector space V, there exists a unique linear map from Z to V. Show that Z is isomorphic to the trivial vector space.

   (c) Let V be a vector space. Show that there exists a unique linear map from V to the trivial vector space.

   (d) Let Z be a vector space such that, for any vector space V, there exists a unique linear map from V to Z. Show that Z is isomorphic to the trivial vector space.

4. (a) Let $T : V \to W$ be a linear map. Show that $\ker(T)$, together with its inclusion $i : \ker(T) \to V$, satisfies the following property: if $S : U \to V$ is a linear map such that $T \circ S = 0$, then there exists a unique linear map $R : U \to \ker(T)$ such that $S = i \circ R$.

   (b) Let K be a vector space and $j : K \to V$ a linear map satisfying the same property as in part (a). (That is, if $S : U \to V$ is a linear map such that $T \circ S = 0$, then there exists a unique linear map $R : U \to K$ such that $S = j \circ R$.) Show that there exists a unique isomorphism from K to $\ker(T)$.