

CHIP FIRING EXERCISES 5

- (1) Let G be a graph of genus g , and let D be a divisor on G of degree 2 and rank 1. Compute the rank of mD for all integers m . Conclude that $K_G \sim (g-1)D$.
- (2) Let G be a simple bipartite graph, and let D be the sum of the vertices of a single color. Show that $\text{rk}(D) \geq 1$.
- (3) Let G be a graph of genus g . Recall that a Weierstrass point is a ramification point of the canonical divisor K_G .
 - (a) Prove that a vertex v is a Weierstrass point if and only if $\text{rk}(gv) \geq 1$.
 - (b) Give an example of a graph of genus at least 2 with no Weierstrass points.
 - (c) Give an example of a graph such that every vertex is a Weierstrass point.