Ex: If c is a constant,

$$\lim_{x \to a} c = c$$

Ex: $$\lim_{x \to a} x = a$$

Limit Laws

Let $f(x)$, $g(x)$ be functions and suppose $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} g(x)$ exists.

Then:

1) $$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x).$$

2) $$\lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} f(x)$$

3) $$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

4) If $\lim_{x \to a} g(x) \neq 0$, then

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

Ex: $\lim_{x \to 5} [x^2 + 3x - 4]$

$$= \underbrace{\lim_{x \to 5} [x^2] + \lim_{x \to 5} [3x] + \lim_{x \to 5} [-4]}_{\text{by (1)}}$$

$$= \underbrace{\left(\lim_{x \to 5} [x]\right)^2}_{\text{by (3)}} + \underbrace{3 \cdot \lim_{x \to 5} [x]}_{\text{by (2)}} + \lim_{x \to 5} [-4]$$

$$= 5^2 + 3 \cdot 5 + (-4)$$

$$= 25 + 15 - 4 = 36.$$

This example shows that, if $p(x)$ is a polynomial, then $\lim_{x \to a} p(x) = p(a)$.

If $p(x)$ and $q(x)$ are polynomials <u>AND</u> $q(a) \neq 0$, then

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}.$$

Def: A function $f(x)$ is continuous at a number a if:

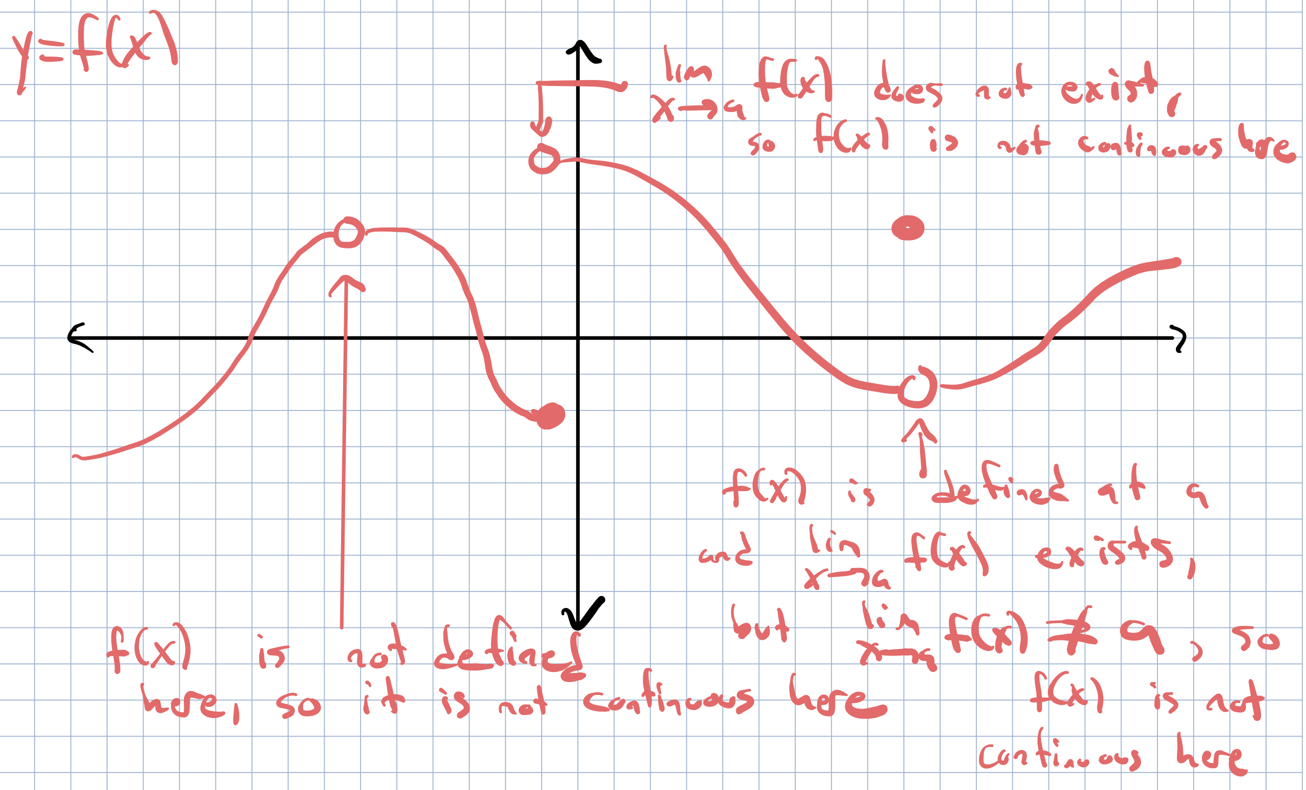
1) $f(x)$ is defined at a
2) $\lim_{x \to a} f(x)$ exists
3) $\lim_{x \to a} f(x) = f(a)$.

A function $f(x)$ is called continuous if it is continuous at a for every a in its domain.

The following types of functions are continuous where they are defined:

- polynomials
- rational functions
- exponentials
- logarithms
- absolute value
- trig functions

Another way to think about continuous functions.



A function is continuous if there are no holes or jumps or breaks in the graph. You can graph the function without lifting your pencil off the paper.

Ex: Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x+b & \text{if } x \geq 2 \end{cases}.$$

For what value of b is $f(x)$ continuous?

Because x^2 and $3x+b$ are continuous functions, $f(x)$ is continuous at every a <u>except</u> possibly $a=2$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} [x^2] = 2^2 = 4$$

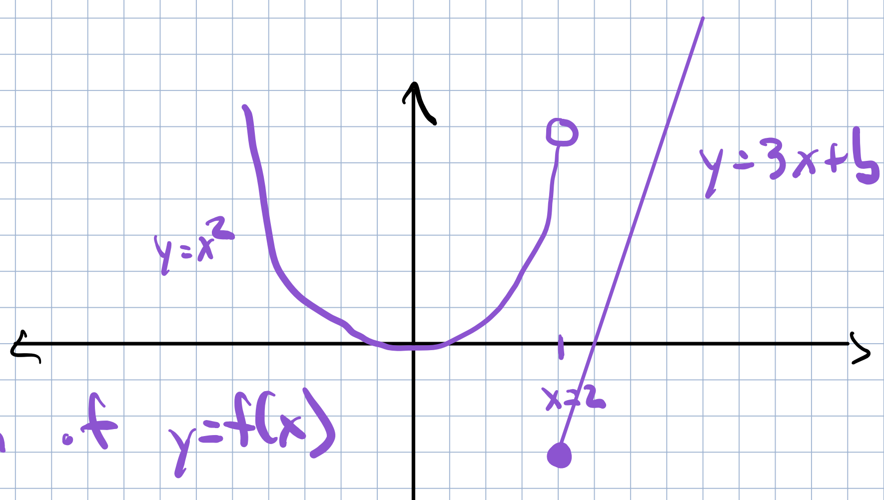
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} [3x+b] = 6+b$$

In order for $\lim_{x\to 2} f(x)$ to exist, we need

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$4 = 6+b$$

$$-2 = b.$$



graph of $y=f(x)$

Want to choose b so that the ○ lines up w/ ●

○ is at $2^2=4$ ● is at $3\cdot 2+b$
$=6+b$

$4 = 6 + b$ $\qquad -2 = b$

Intermediate Value Theorem

Let $f(x)$ be a function that is continuous on $[a, b]$, and let z be a number between $f(a)$ and $f(b)$.

Then there exists a number c between a and b such that $f(c) = z$.