# Derivatives

Take the (optional) anonymous course survey on Canvas under **Quizzes**

I drive from here to Cincinnati (75 miles). It takes me an hour and a half.

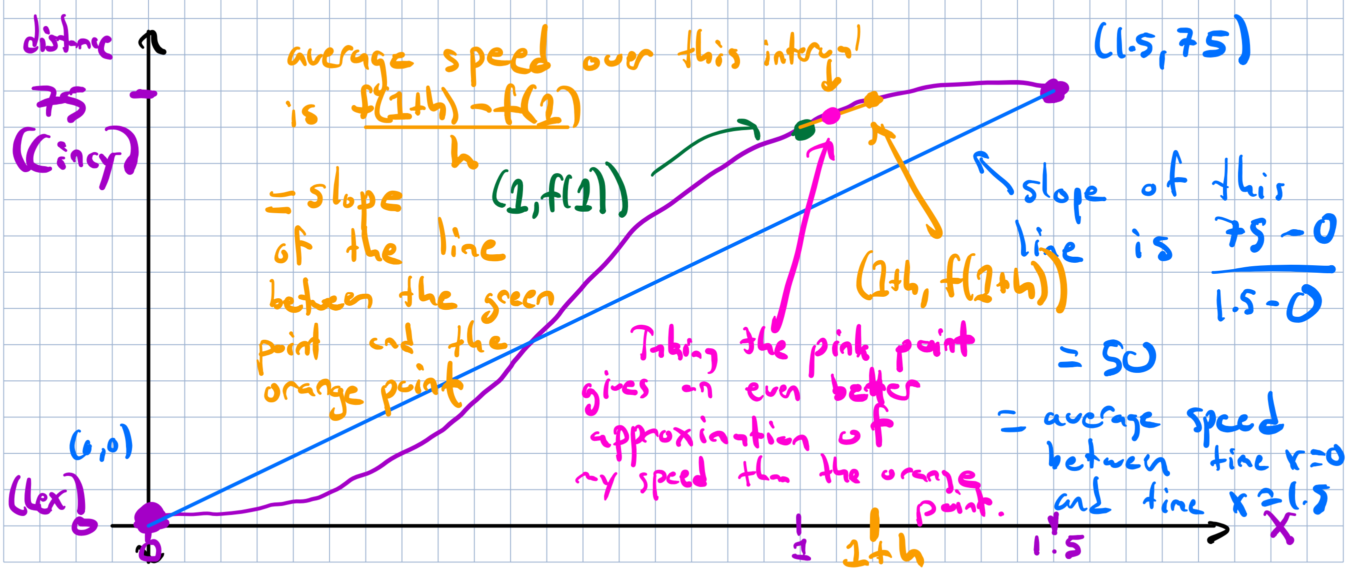
What was my average speed?

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{75 \text{ miles}}{1.5 \text{ hours}}$$

$= 50$ mph.

$x$ = time in hours

$f(x)$ = distance traveled after $x$ hours



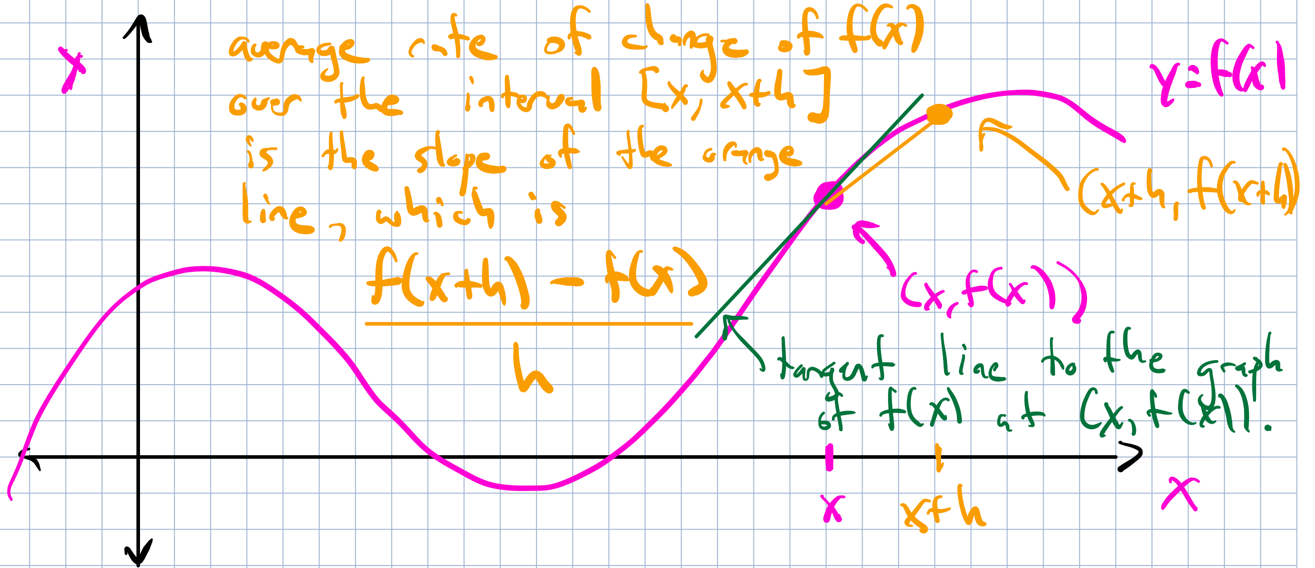
Instead of computing my <u>average</u> speed, what if I want to compute my <u>instantaneous</u> speed?

If I compute my average speed over a smaller interval of time, then I obtain a better approximation for my instantaneous speed.

Put all this together.

I have a function $f(x)$.

I want to know the rate at which that function is changing at a certain value of $x$.




Instantaneous rate of change of $f(x)$ at $x$ is

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

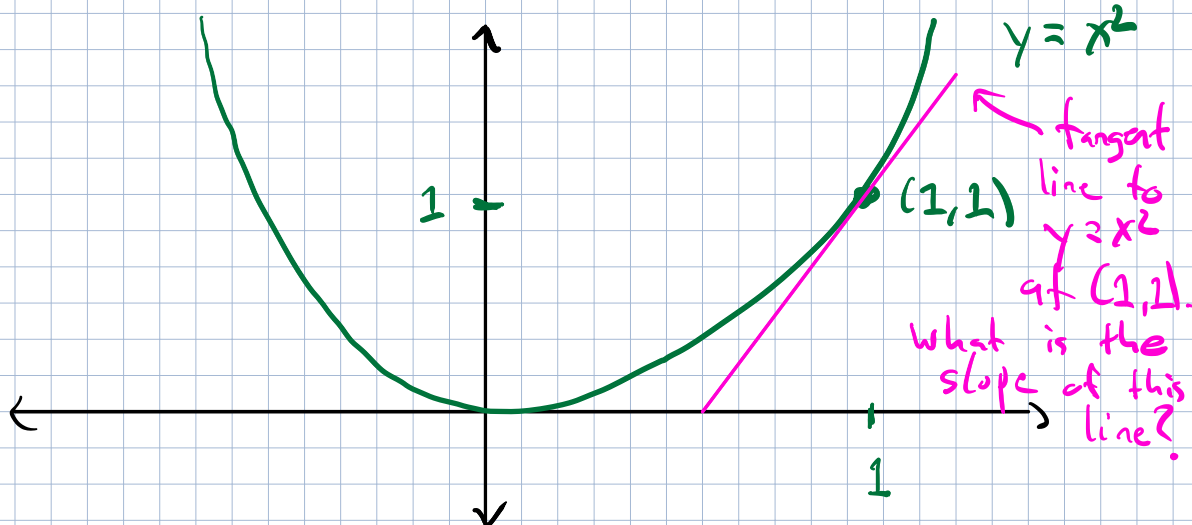
$= $ slope of the tangent line to the graph $y=f(x)$ at $(x, f(x))$.

The **derivative** of $f(x)$ at $x$ is

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.}$$

The derivative of $y$ with respect to $x$ is also written $\frac{dy}{dx}$.

Ex: Let $f(x) = x^2$. Compute $f'(1)$.




$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h} = \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h} = \lim_{h \to 0} 2 + h$$

$$= 2 + 0 = 2$$

What is the tangent line to $y = x^2$ at $(1,1)$.

A line w/ slope 2 that goes through $(1,1)$.

$y = 2x + b$

$1 = 2 \cdot 1 + b$

$-1 = b$

$$y = 2x - 1$$