Final Projects are graded

Canvas deleted all my comments, so if you have questions about your grade, please ask

Final Exam Wednesday, May 4 10:30 AM - 12:30 PM

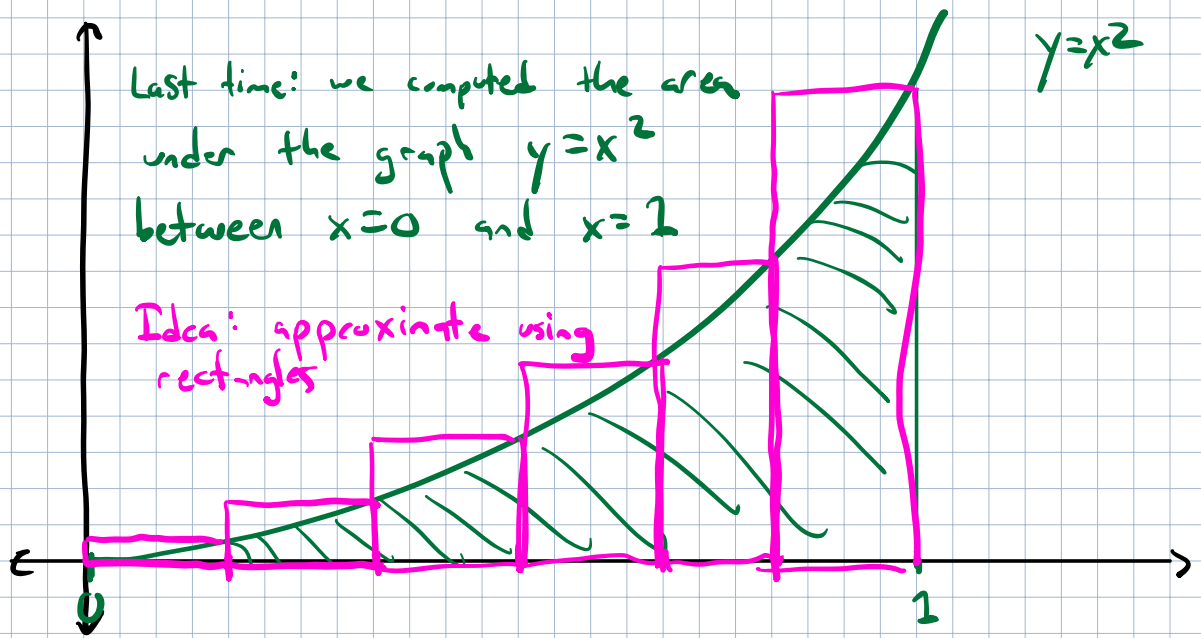
Final will be cumulative

Final course grades will be submitted to the registrar by the end of the day on Wednesday, May 4th.

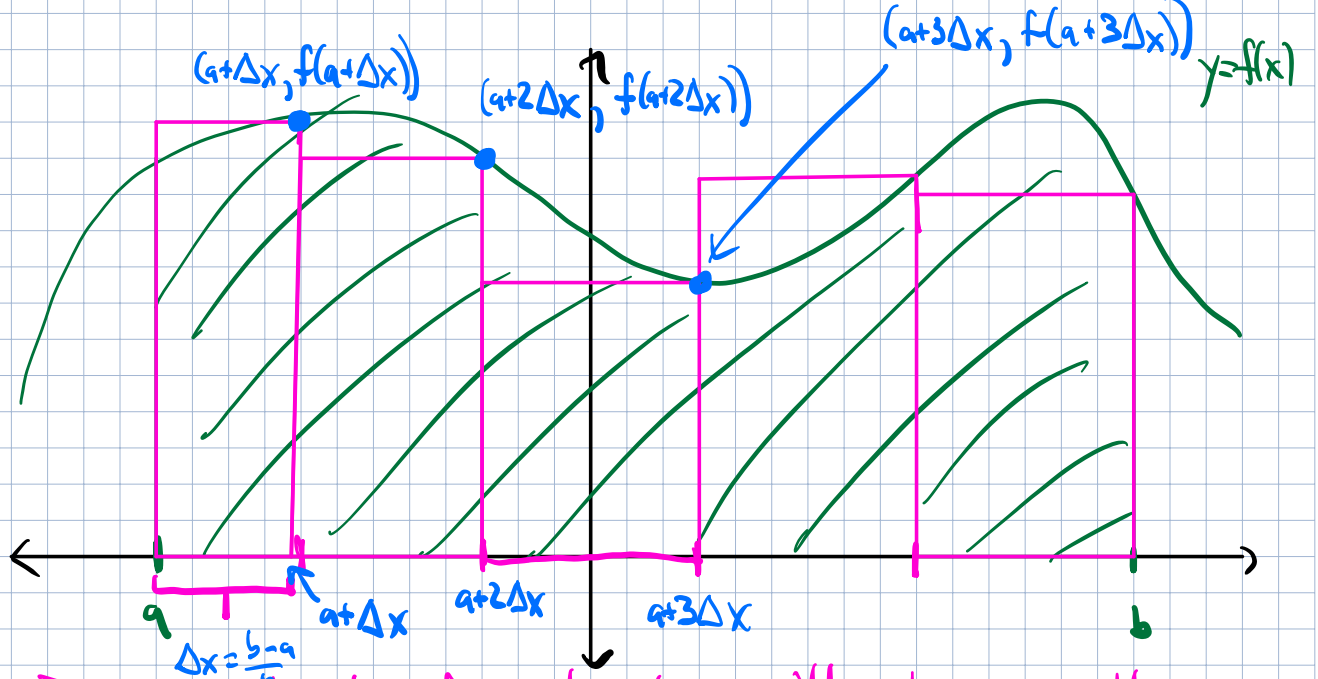
→ If you miss the final or have any other outstanding coursework, you need to contact me on or before May 4

I will be dropping the 3 lowest quiz grades

Computing Areas



Today: compute the area under the graph $y = f(x)$ between $x = a$ and $x = b$.



Draw a bunch of rectangles with base on the x-axis and upper right corner on the graph

# of rectangles $= n$

width of each rectangle $= \Delta x = \dfrac{b-a}{n}$

Area of the rectangles

$= f(a+\Delta x)\cdot\Delta x + f(a+2\Delta x)\cdot\Delta x + f(a+3\Delta x)\cdot\Delta x$

$+ \cdots + f(a+n\cdot\Delta x)\cdot\Delta x$

$= \left[f(a+\Delta x) + f(a+2\Delta x) + f(a+3\Delta x) + \cdots + f(a+n\Delta x)\right]\cdot\Delta x$

Digression About Summation Notation

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

(n: what's the last value; a_i: what do you plug i in to; 1: what's the first value of i to plug in)

Ex: $\sum_{i=1}^{6} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

$= 1 + 4 + 9 + 16 + 25 + 36$

$= 91$

Ex: $\sum_{i=3}^{5} (i^3 - i) = (3^3 - 3) + (4^3 - 4) + (5^3 - 5)$

$= (27 - 3) + (64 - 4) + (125 - 5)$

$= 24 + 60 + 120 = 204$

$= [f(a+\Delta x) + f(a+2\Delta x) + f(a+3\Delta x) + \cdots + f(a+n\Delta x)] \cdot \Delta x$

$= \sum_{i=1}^{n} f(a + i\Delta x) \cdot \Delta x$ ← area of the n rectangles

To compute the actual area, take the limit as the # of rectangles goes to infinity

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\,\Delta x$$

Riemann sum

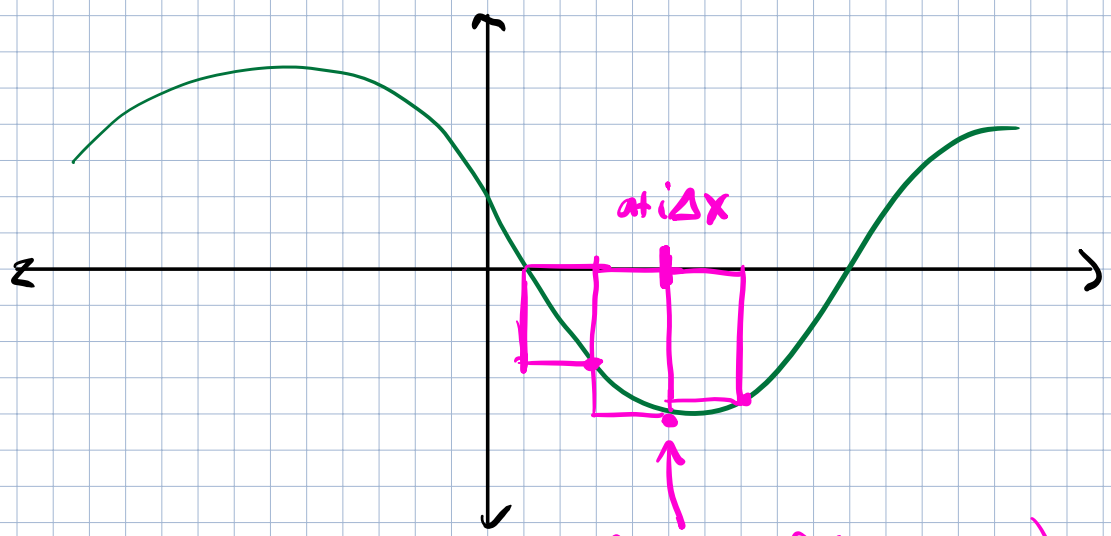
$\int_a^b f(x)\,dx$ is called the definite integral of $f(x)$ from a to b.

What does the definite integral compute?

$\int_a^b f(x)\,dx$ is the signed area under the graph $y=f(x)$ between $x=a$ and $x=b$.

2 caveats:

(1) What if the graph dips below the x-axis?



The "height" of this rectangle is $f(a+i\Delta x) < 0$

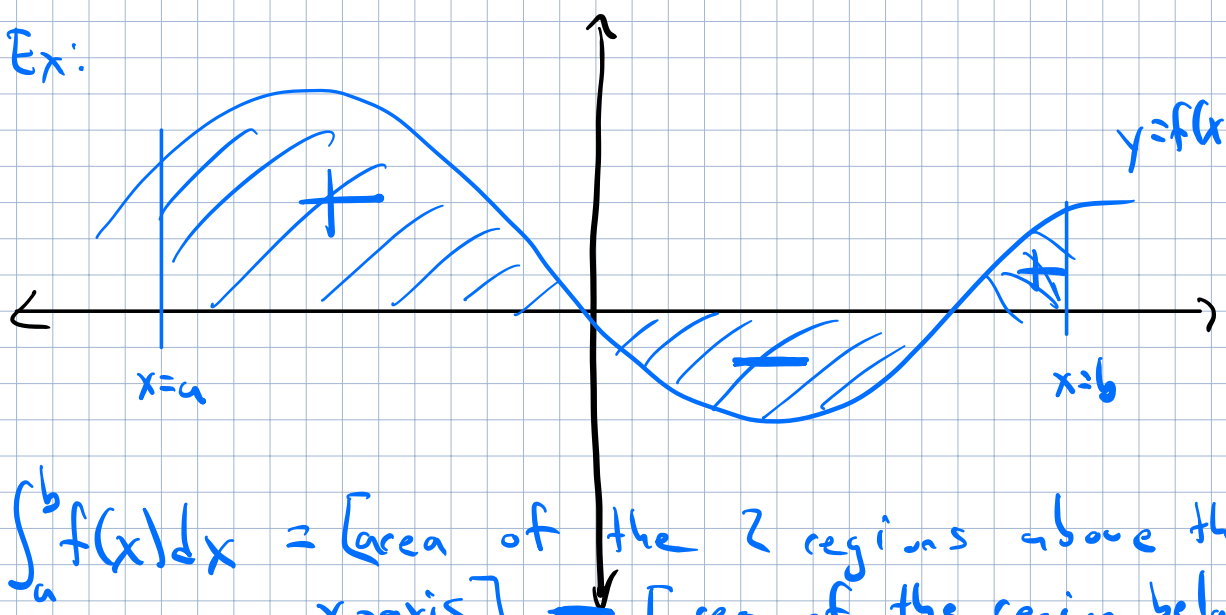
(2) What if $b < a$?

The rectangles have "width" $\Delta x = \frac{b-a}{n} < 0$

Rectangles can have negative "height" if $f(x) < 0$

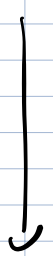
can have negative "width" if $b < a$.

Ex:



$\int_a^b f(x)\,dx$ = [area of the 2 regions above the x-axis] − [area of the region below the x-axis]

Ex: Compute $\int_{-1}^{2} 2x\,dx$.



$$\int_{-1}^{2} 2x\,dx = 4 - 1$$
$$= 3$$