Exam 1 IN CLASS February 2^{nd} (a week from Wednesday)

Format: 6 multiple choice questions
2 short answer questions
calculators allowed

What's on it?

- Elementary Functions / Composition and Inverses
- Exponential Functions
- Logarithms
- Semi-log and Double-log Plots
- Exponential Growth
- Sequences and Limits of Sequences

Sequences

exponential sequences: $a_n = C \cdot R^n$.

$a_0 = C$, $a_1 = C \cdot R$, $a_2 = C \cdot R^2$, $a_3 = C \cdot R^3$, ...

A sequence is a function that assigns a number a_n to every nonnegative integer n.

Ex: $a_n = \frac{1}{n+1}$

$a_0 = \frac{1}{0+1} = \frac{1}{1} = 1$

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{1}{2+1} = \frac{1}{3}$$

$$a_3 = \frac{1}{3+1} = \frac{1}{4}$$

$$a_4 = \frac{1}{4+1} = \frac{1}{5}$$

$$\vdots$$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$$

Ex: $a_n = (-1)^n$

$$a_0 = (-1)^0 = 1$$

$$a_1 = (-1)^1 = -1$$

$$a_2 = (-1)^2 = (-1)(-1) = 1$$

$$a_3 = (-1)^3 = (-1)(-1)(-1) = -1$$

$$1, -1, 1, -1, 1, -1, 1, -1, \ldots$$

Ex: $a_n = \dfrac{2n-3}{5+n}$

$$a_0 = \frac{2\cdot 0 - 3}{5+0} = \frac{-3}{5}$$

$$a_1 = \frac{2\cdot 1 - 3}{5+1} = \frac{-1}{6}$$

$$a_2 = \frac{2\cdot 2-3}{5+2} = \frac{1}{7}$$

$$a_3 = \frac{2\cdot 3-3}{5+3} = \frac{3}{8}$$

$$a_4 = \frac{2\cdot 4-3}{5+4} = \frac{5}{9}$$

$$-\frac{3}{5}, -\frac{1}{6}, \frac{1}{7}, \frac{3}{8}, \frac{5}{9}, \ldots$$

The 1000^{th} term in the sequence is

$$a_{1000} = \frac{2\cdot 1000-3}{5+1000} = \frac{1997}{1005}$$

$$\approx 1.987\ldots$$

Def: If, as n gets larger and larger, the values a_n get arbitrarily close to some fixed number L, then we say that L is the limit of the sequence as n goes to infinity, and we write $L = \lim_{n\to\infty} a_n$.

Ex: $a_n = \frac{1}{n+1}$.

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

$a_{1000} = \frac{1}{1000} = 0.001$

$a_{1,000,000} = \frac{1}{1,000,000} = 0.000001$

As n gets larger and larger, the values a_n get arbitrarily close to 0.

$$\lim_{n\to\infty} a_n = 0.$$

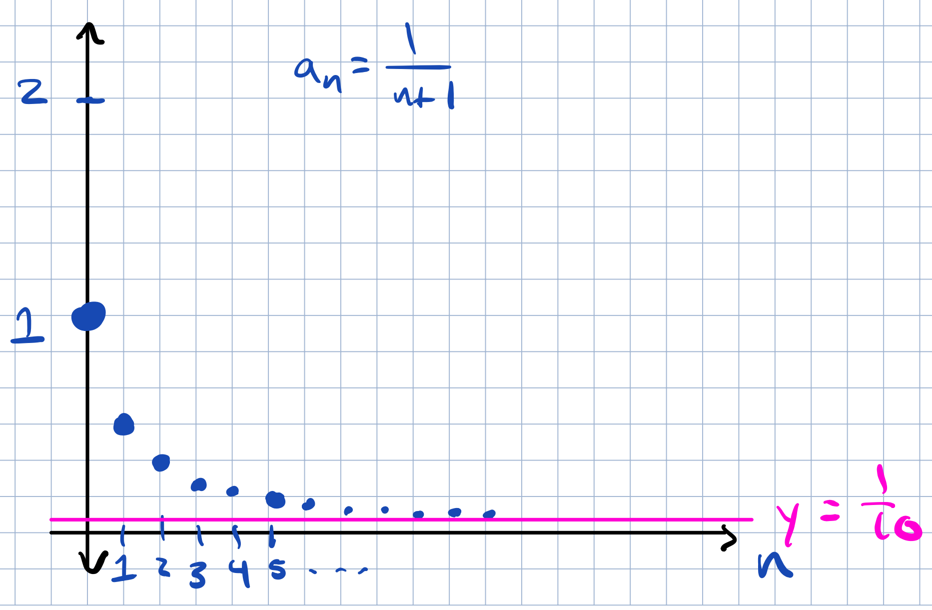
Ex: $a_n = \dfrac{2n-3}{5+n}$.

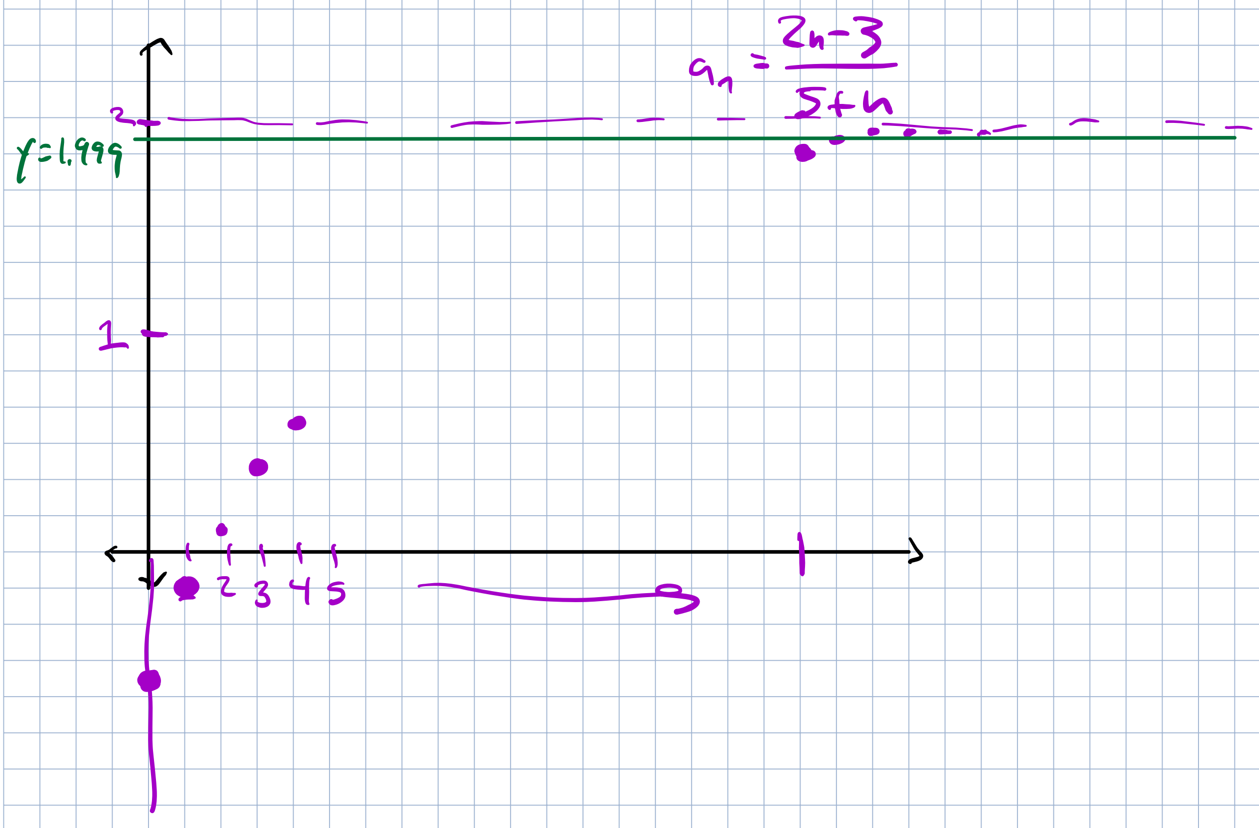
$$a_{1000} = \frac{1997}{1005} \approx 1.987\ldots$$

$$a_{1{,}000{,}000} = \frac{1999997}{1000005} \approx 1.999987$$

As n grows, a_n gets as close as you like to 2.

So $\displaystyle\lim_{n\to\infty} a_n = 2$.





WARNING! Not all sequences have limits!

Ex: $a_n = 2^n$.

$$2, 2, 4, 8, 16, 32, 64, 128, \ldots$$

As n gets larger and larger, the value a_n also gets larger and larger.

$$\lim_{n\to\infty} 2^n \text{ does not exist.}$$

There is no number that the values approach.

$$\lim_{n\to\infty} 2^n = \infty.$$

Ex: $a_n = (-1)^n$

$1, -1, 1, -1, 1, -1, 1, -1, \ldots$

No matter how far out you go in this sequence, the terms in the sequence do not get close to any fixed number.

There's always a 1 and there's always a -1.

$$\lim_{n \to \infty} (-1)^n \text{ does not exist.}$$