

## Assignment 12

1. Let  $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$ .

- Find the projection of  $\vec{b}$  onto the column space of  $A$ .
  - Use the projection in (a) to find the least squares solution to  $A\vec{x} = \vec{b}$ .
  - Find the least squares solutions to  $A\vec{x} = \vec{b}$  using the transpose of  $A$ .
2. Give the least squares solutions to the system

$$\begin{aligned} x - y &= 4 \\ x - y &= 6 \end{aligned}$$

3. For the data (1,0), (2,1), (4,2), (5,3) find the equation of best fit of the form
- $ax + b$ ,
  - $ax^2 + bx$
4. Suppose radioactive substances  $A$  and  $B$  have decay constants of .02 and .07 respectively. If a mixture of these two substances at time  $t = 0$  contains  $M_A$  grams of  $A$  and  $M_B$  grams of  $B$ , then a model for the total amount of  $y$  of the mixture present at times  $t$  is

$$y = M_A e^{-.02t} + M_B e^{-.07t}$$

Suppose the initial amounts  $M_A$  and  $M_B$  are unknown, but a scientist is able to measure the total amount present at several times and records the following points  $(t, y)$ : (10, 21.34), (11, 20.68), (12, 20.05), (14, 18.87), (15, 18.30)

- What least squares problem do you need to solve to find  $M_A$  and  $M_B$ ?
- (Use technology!) Solve this least squares problem.