

Assignment 9

1. Diagonalize the matrix

$$\begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$$

if possible. (Hint: 1 and 2 are eigenvalues.)

2. Construct a nondiagonal 2×2 matrix that is diagonalizable but not invertible.
3. Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$, $\vec{b}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$. Find a matrix A so that $[T(\vec{v})]_{\mathcal{B}} = A[\vec{v}]_{\mathcal{B}}$.
4. Let $T(\vec{v}) = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix} \vec{v}$. Find a basis \mathcal{B} for \mathbb{R}^2 so that the matrix of T is diagonal with respect to \mathcal{B} .
5. Find the eigenvalues and eigenvectors of $\begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$.
6. Suppose A is an $n \times n$ real matrix and $A\vec{v} = \lambda\vec{v}$ for some complex number $\lambda = (a + ib)$.
- (a) Write $A(\operatorname{Re}(\vec{v}) + i\operatorname{Im}(\vec{v}))$ using a , b , $\operatorname{Re}(\vec{v})$, and $\operatorname{Im}(\vec{v})$.
- (b) Show $A(\operatorname{Re}(\vec{v})) = a\operatorname{Re}(\vec{v}) + b\operatorname{Im}(\vec{v})$ and $A(\operatorname{Im}(\vec{v})) = -b\operatorname{Re}(\vec{v}) + a\operatorname{Im}(\vec{v})$.