MA 665 EXERCISES 2

(1) In a category with a zero object, show that the zero morphism from an object A to an object B is unique. In other words, if the category has two zero objects Z and Z', then the composites $A \to Z \to B$ and $A \to Z' \to B$ are the same.

(2) Show that the product of two objects A and B in a category $\mathcal{C}$ is *unique up to unique isomorphism*. In other words, if (C, p_1, p_2) and (D, q_1, q_2) are both products of A and B, then there is a unique isomorphism $g : C \to D$ such that $p_1 = q_1 \circ g$ and $p_2 = q_2 \circ g$.

(3) Formulate and prove the statement that the kernel of a morphism is unique up to unique isomorphism.