MA 665 EXERCISES 6

(1) Show that the following are equivalent.
  (a) B is an injective R-module.
  (b) $\mathrm{Ext}^i_R(A, B) = 0$ for all $i \neq 0$ and all A.
  (c) $\mathrm{Ext}^1_R(A, B) = 0$ for all A.

(2) Let R be an integral domain with field of fractions K. Show that $\mathrm{Tor}^R_1(K/R, B)$ is the torsion submodule of B for every R-module B.

(3) Let p be a prime, suppose p^2 divides m, let $R = \mathbb{Z}/m\mathbb{Z}$ and $B = \mathbb{Z}/p\mathbb{Z}$. Show that

$$0 \to \mathbb{Z}/p\mathbb{Z} \hookrightarrow \mathbb{Z}/m\mathbb{Z} \xrightarrow{p} \mathbb{Z}/m\mathbb{Z} \xrightarrow{m/p} \mathbb{Z}/m\mathbb{Z} \xrightarrow{p} \mathbb{Z}/m\mathbb{Z} \xrightarrow{m/p} \cdots$$

is an infinite periodic injective resolution of B. Prove that

$$\mathrm{Ext}^n_{\mathbb{Z}/m\mathbb{Z}}(\mathbb{Z}/p\mathbb{Z}, \mathbb{Z}/p\mathbb{Z}) \cong \mathbb{Z}/p\mathbb{Z}$$

for all n.