

AP Calculus BC

Calculator Lab

Applications of the Definite Integral

Arclength of a Curve

Most of the time when we try to compute the arclength of a curve from the formula derived in the text, we encounter functions which do not have elementary antiderivatives.

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Quite often we encounter elliptic integrals in different forms. There are tables of such integrals and any good computer algebra system will be able to numerically evaluate these integrals. We can use the calculator similarly. Our integrator, or the approximation using Simpson's Rule, will give us a relatively close answer for most nice functions.

A golf ball is hit with a driver and, neglecting almost all physical forces except gravity, follows the path

$$y = -\frac{x^2}{600} + \frac{x}{2}$$

The ball is on the ground at $x = 0$ and at $x = 300$. We will assume that the ball did not roll when it hit the ground. We would normally say that the drive was 300 yards long, which is the distance along the ground that the ball travelled. The actual distance that the ball travelled is given by the arclength of the parabola



The arclength is given by the integral

$$\int_0^{300} \sqrt{1 + [y']^2} dx = \int_0^{300} \sqrt{1 + (-x/300 + 1/2)^2} dx$$

Problems:

1. Use the integrator on your calculator or Simpson's rule to evaluate this integral. How far did the ball travel?

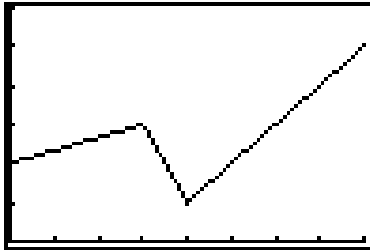


Figure 1 Figure 2

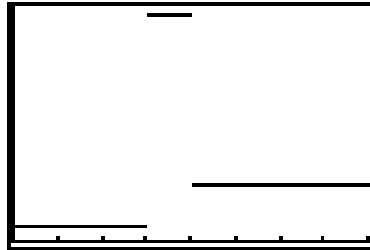
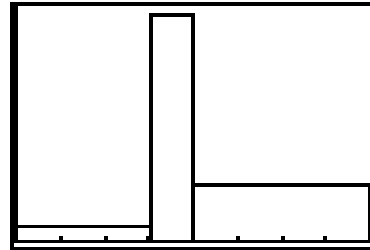


Figure 3



2. The graph of the function $f(x)$ in Figure 1 consists of three line segments for $0 \leq x \leq 8$. Each mark along the x -axis in Figures 1, 2, and 3, and each mark along the y -axis in Figure 1 is 1 unit.
- What is the total length of the graph?
 - What is $f'(x)$ for $0 < x < 3$?
 - What is $f'(x)$ for $3 < x < 4$?
 - What is $f'(x)$ for $4 < x < 8$?
 - Figure 2 above shows the graph of $\sqrt{1 + [f'(x)]^2}$. What are the y -coordinates of the three lines that form its graph?
 - Find the areas of the three rectangles in Figure 3.
 - How are the answers to parts (a) and (f) related?

Lab Report

The report for this lab consists of your graphs and your answers to all questions. Just copy your graphs as best you can from the calculator. Be sure that your answers are in complete sentences and that you show your work.