Let C denote the circle of radius r centered at the origin. Generate the sphere of radius r by rotating the circle about the x-axis. The function defining this circle is

$$f(x) = \sqrt{r^2 - x^2} \ ,$$

so

$$f'(x) = -\frac{x}{\sqrt{r^2 - x^2}}$$

and

$$1 + \left[f'(x) \right]^2 = 1 + \left[-\frac{x}{\sqrt{r^2 - x^2}} \right]^2 = \frac{r^2}{r^2 - x^2} \ .$$

The surface area of this solid is given by:

$$SA = 2 \int_0^r 2\pi f(x) \sqrt{1 + \left[f'(x) \right]^2} \, dx$$

$$= 4\pi \int_0^r \sqrt{r^2 - x^2} \sqrt{\frac{r^2}{r^2 - x^2}} \, dx$$

$$= 4\pi \int_0^r r \, dx = 4\pi r^2$$