Assignment 2: Solutions

1. *Divide* 3^{175} *by* $50!$. *What is the numerator of the resulting reduced fraction? How many factors of 3 does the factorization of this integer contain? Use* ***Maple****'s* `numer` *command to obtain the numerator. Then click the right mouse button on the result and choose the option* Integer Factors *from the pop-up menu.*

```
> 3^175/50!;
```

9989689095948428268966921126195809393034773710522520293009978943 1\
47202723/969186304754053024418147147150552064851968000000000000

```
> numer(%);
```

99896890959484282689669211261958093930347737105225202930099789431\
47202723

```
> ifactor(%);
```

$$(3)^{153}$$

2. *Determine the time it takes for* ***Maple*** *to compute* $5000!$. *Which takes more time, computing* $5000!$ *or* 5000^{5000}*?*

```
> restart:
> showtime();
> 5000!:
time = 0.06, bytes = 493442
> 5000^5000:
time = 0.01, bytes = 41082
> off;
```

3. *i) Compute* $\log(x)$ *(the base 10 logarithm) of the following numbers:*

$$x = 56,\ 123,\ 5120,\ 98765.$$

Use ***Maple****'s* `evalf` *command to obtain numerical results.*

```
> restart;
> L := [56, 123, 5120, 98765];
```

$$L := [56,\ 123,\ 5120,\ 98765]$$

```
> map(log10,L);
```

$$[\frac{\ln(56)}{\ln(10)}, \frac{\ln(123)}{\ln(10)}, \frac{\ln(5120)}{\ln(10)}, \frac{\ln(98765)}{\ln(10)}]$$

```
> map(evalf,%);
```

$$[1.748188027,\ 2.089905111,\ 3.709269961,\ 4.994603068]$$

ii) *Now, compute the largest integer less than or equal to* $\log(x)$ *for these numbers. Use* ***Maple****'s* `floor` *function. The mathematical notation for the floor function is* $\lfloor x \rfloor$.

```
> map(floor,%);
```

$$[1, 2, 3, 4]$$

(a) *What is the relationship between* $\lfloor \log(x) \rfloor$ *and the number of digits of the number* x*?*

Note that this is called the mantissa of a number. This is one less than the number of digits in x.

iv) *What is the number of digits of the huge integer* $2^10000000$ *(2 to the power ten million)? Use* ***Maple****'s* `floor` *function and* `log10` *commands and remember that* $\log(2^k) = k \cdot \log(2)$.

```
> floor(10000000*log10(2));
```

$$3010299$$

Thus there are 3010300 digits in $2^{10000000}$.

4. *The* ***Maple*** *procedure* `history` *makes the percentage symbols* `%`, `%%`, *and* `%%%` *less useful, if not superfluous. Use* ***Maple****'s help facility to check on how precisely* `history` *functions. Then find out if* `%`, `%%`, *and* `%%%` *are still working as before.*

```
> ?history
```

The `history` function is used to maintain a history of all results computed within a ***Maple*** session, beyond what is available with the ditto operators %, %%, and %%%. After loading the history function, type `history()`; to initiate the history mechanism. You will be prompted for input by the prompt `O1 :=` and for successive statements by the prompts `O2 :=`, `O3 :=`, and so on. Any ***Maple*** statement entered is evaluated and displayed normally. The result is assigned to the global variables `O1`, `O2`, ... which may be referred to later, thus providing a history mechanism. The ditto operators will not work as before.

5. *The first step in understanding the proper use of* ***Maple*** *packages such as* `student` *is the issuing of instructions like* `?with` *and* `?student`. *Tell me how a single package procedure cam be used without having to load the entire package.*

```
> ?with
```

When `with` is called with the name of a package as its argument, it *imposes* the names exported by the package on the global namespace. The effect of this is to make these exported names available to the interactive user at the top level of ***Maple***'s interaction loop. For example, to access the procedure `combinat[powerset]` you must type

```
> combinat[ 'powerset' ]( { 1, 2 } );
```

$$\{\{\}, \{1, 2\}, \{2\}, \{1\}\}$$

However, using `with` allows you to use the name `powerset` directly, without specifying the package to which that routine belongs.

```
> with( combinat );
```

```
Warning, the protected name Chi has been redefined and unprotected
```

[Chi, *bell*, binomial, *cartprod*, *character*, *choose*, *composition*, *conjpart*, *decodepart*, *encodepart*, *fibonacci*, *firstpart*, *graycode*, *inttovec*, *lastpart*, *multinomial*, *nextpart*, *numbcomb*, *numbcomp*, *numbpart*, *numbperm*, *partition*, *permute*, *powerset*, *prevpart*, *randcomb*, *randpart*, *randperm*, *stirling1*, *stirling2*, *subsets*, *vectoint*]

```
> powerset( { 'a', 'b' } );
```

$$\{\{\}, \{b\}, \{a\}, \{a, b\}\}$$

6. Verify that

$$\sum_{i=1}^{\infty} \left\lfloor (\frac{n}{2^i}) \right\rfloor$$

gives the number of factors of 2 occurring in the factorization of $n!$. Remember that $\lfloor x \rfloor$ denotes the floor function applied to x.

Clearly, for i sufficiently large, $\lfloor (\frac{n}{2^i}) \rfloor$ will be zero.

```
> i:='i';
```

$$i := i$$

We define a function f on the set of natural numbers by

```
> f := n -> sum(floor(n/2^i),i=1..n);
```

$$f := n \to \sum_{i=1}^{n} \mathrm{floor}(\frac{n}{2^i})$$

Now, we want to verify for a number of integers that $n!/2^{f(n)}$ is odd. That means that $f(n)$ accounts for all of the factors of 2 in $n!$.

```
> for j from 2 to 25 do j!/(2^f(j)) od;
```

1
3
3
15
45
315
315
2835
14175
155925
467775
6081075

$$42567525$$
$$638512875$$
$$638512875$$
$$10854718875$$
$$97692469875$$
$$1856156927625$$
$$9280784638125$$
$$194896477400625$$
$$2143861251406875$$
$$49308808782358125$$
$$147926426347074375$$
$$3698160658676859375$$

Note that each of these is odd.

```
> for j from 1 to 4 do floor(25/2^j) od;
```

$$12$$
$$6$$
$$3$$
$$1$$

$\lfloor n/2^j \rfloor$ tells us the number of multiples of 2^j that there are that are less than n.