Appendix E

Quartic Formula

$$r_1 = -\frac{a}{4} - \frac{1}{2}\left[\frac{a^2}{4} - \frac{2b}{3} + \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}}$$

$$-\frac{1}{2}\left[\frac{a^2}{2} - \frac{4b}{3} - \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} - \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}}}}\right]^{\frac{1}{2}}$$

$$r_2 = -\frac{a}{4} - \frac{1}{2}\left[\frac{a^2}{4} - \frac{2b}{3} + \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}}$$

$$+\frac{1}{2}\left[\frac{a^2}{2} - \frac{4b}{3} - \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} - \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{\sqrt[3]{2}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54}\right)^{\frac{1}{3}}}}\right]^{\frac{1}{2}}$$

$$r_3 = -\frac{a}{4}+\frac{1}{2}\left[\frac{a^2}{4} - \frac{2b}{3} + \frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}}\right.$$
$$\left. + \left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}}$$
$$-\frac{1}{2}\left[\frac{a^2}{2} - \frac{4b}{3} - \frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}} - \right.$$
$$\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}} -$$
$$\left.\frac{-a^3+4ab-8c}{4\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}}+\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}}}}\right]^{\frac{1}{2}}$$

$$r_4 = -\frac{a}{4}+\frac{1}{2}\left[\frac{a^2}{4}-\frac{2b}{3}+\frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}}\right.$$
$$\left.+\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}}$$
$$+\frac{1}{2}\left[\frac{a^2}{2}-\frac{4b}{3}-\frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}}-\right.$$
$$\left.\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}}-\right.$$
$$\left.\frac{-a^3+4ab-8c}{4\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{\sqrt[3]{2}(b^2-3ac+12d)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}\right)^{\frac{1}{3}}}+\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4(b^2-3ac+12d)^3+(2b^3-9abc+27c^2+27a^2d-72bd)^2}}{54}\right)^{\frac{1}{3}}}}\right]^{\frac{1}{2}}$$