ASSIGNMENT 2

11-September-2006

1. Let s be a nonzero rational number and t be irrational.
   (a) Prove that $s - t$ is irrational.
   (b) Prove that st is irrational.
   (c) Prove that s/t is irrational.

2. Although $\sqrt{2} + \sqrt{3}$ does not equal the square root of an integer, $\sqrt{27} + \sqrt{48}$ does.
   (a) What integer's square root equals $\sqrt{27} + \sqrt{48}$?
   (b) Find another different example like $\sqrt{27} + \sqrt{48}$.
   (c) Find every set of different positive integers p, q, and r all less than 100 such that p and q are not perfect squares and $\sqrt{p} + \sqrt{q} = \sqrt{r}$.

3. Suppose that p and q are positive integers and that $a = \frac{p}{q}$. **Explain** why long division of p by q results in the decimal representation of a.
   [Hint: It is enough to explain why the decimal $d = [D, d_1, d_2, d_3, \ldots, d_k, \ldots]$ produced by long division satisfies
   $$D + \frac{d_1}{10} + \frac{d_2}{10^2} + \cdots + \frac{d_k}{10^k} \leq a \leq D + \frac{d_1}{10} + \frac{d_2}{10^2} + \cdots + \frac{d_k}{10^k} + \frac{1}{10^k}$$
   for all $k \in \mathbb{N}$.]

4. Prove the following properties of any complex number z.
   (a) $\text{Re}[z] = \frac{z+\bar{z}}{2}$
   (b) $\text{Im}[z] = \frac{z-\bar{z}}{2i}$

5. Suppose that $[r, \theta]$ is a point z in the complex plane, $x = r\cos\theta$ and $y = r\sin\theta$. If $r' = -r$ and $\theta' = \theta + \pi(2n+1)$, where n is an integer, prove that $[r', \theta']$ determines the same rectangular representation (x, y) of z.

6. Track the solution set in the complex plane of the quadratic equation $x^2 + bx + 2 = 0$ as the value of the real coefficient b varies.

7. Find the fifth roots of unity. Find the fifth roots of i. Plot them in the complex plane. How are they related?

8. Find $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$, and z_1/z_2.
   (a) $z_1 = -2 + 8i$, $z_2 = -2 - 8i$.
   (b) $z_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $z_2 = (1, 0)$.
   (c) $z_1 = [3, 225°]$, $z_2 = 7(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$.
   (d) z_1 and z_2 are the solutions to $x^2 + x + 1 = 0$.