ASSIGNMENT 5

02-October-2006

1. Use the limit convergence test to decide whether the following series converge or diverge. Note that you need to know convergence of the p-series.

   (a) Does the series $\sum_{n=1}^{\infty} \frac{n+5}{n^3-2n+3}$ converge or diverge?

   (b) Does the series $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ converge of diverge?

2. (a) What is the actual limit of the sum $\sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n$?

   (b) What is the actual limit of the sum $\sum_{n=10}^{\infty} \left(\frac{3}{4}\right)^n$?

   (c) Does the sum $\sum_{n=1}^{\infty} \frac{2^n+1}{3^n-4}$ converge? What test do you use to determine convergence or divergence?

3. Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges. [HINT: Show that $\sqrt{a_n b_n} \leq a_n + b_n$.]

4. Determine which of the following series converge and justify your answer.

   (a) $\sum_{n=1}^{\infty} \frac{n^4}{2^n}$

   (b) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

   (c) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

   (d) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

   (e) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

5. We have seen that it is often harder to find the value of an infinite sum than to show that it exists. Here are some sums that you can find.

   (a) Calculate $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ and $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n$.

   (b) Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. Compare assignment 2.

   (c) Prove that $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. [*Hint*: Note that $\frac{k-1}{2^{k+1}} = \frac{k}{2^k} - \frac{k+1}{2^{k+1}}$].

   (d) Use (c) to compute $\sum_{n=1}^{\infty} \frac{n}{2^n}$.