ASSIGNMENT 8

29-October-2008
Due 05-November-2008

1. Find the limits of the following sequences. (Note: algebraic tricks required.)

   (a) $a_n = \sqrt{n^2+1} - n$.
   [HINT] Show that $a_n = \frac{1}{\sqrt{n^2+1}+n}$.

   (b) $b_n = \sqrt{n^2+n} - n$.

   (c) $c_n = \sqrt{4n^2+n} - 2n$.

2. Suppose that $\lim x_n = 3$, $\lim y_n = 7$ and that all y_n are nonzero. Determine the following limits:

$$(a)\lim(x_n + y_n) \qquad\qquad (b)\lim \frac{3y_n - x_n}{y_n^2}$$

3. Let $a_1 = 1$ and for $n \geq 1$ let $a_{n+1} = \sqrt{a_n + 1}$.

   (a) List the first five terms of $\{a_n\}$.

   (b) It turns out that $\{a_n\}$ converges. Assume that this is true and show that the limit is $\frac{1}{2}(1+\sqrt{5})$.

4. Let $a_1 = 1$ and $a_{n+1} = \frac{1}{3}(a_n + 1)$ for $n \geq 1$.

   (a) Find a_2, a_3, a_4 and a_5.

   (b) Use induction to show that $a_n > \frac{1}{2}$ for all n.

   (c) Show that $\{a_n\}$ is a nonincreasing sequence.

   (d) Find $\lim a_n$.

5. For each of the following sequences find the $\text{glb}\{a_n\}$, $\text{lub}\{a_n\}$, $\limsup\{a_n\}$, and $\liminf\{a_n\}$.

   (a) $\{(-1)^n\}_{n=0}^{\infty}$

   (b) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

   (c) $\{(-1)^n n\}_{n=0}^{\infty}$

6. Let $\{a_n\}$ and $\{b_n\}$ be the following sequences that repeat in cycles of four.

$$\begin{aligned} \{a_n\} &= \{0,1,2,1,0,1,2,1,0,1,2,1,0,1,2,1,0,\ldots\} \\ \{b_n\} &= \{2,1,1,0,2,1,1,0,2,1,1,0,2,1,1,0,2,\ldots\} \end{aligned}$$

   Find

   (a) $\liminf a_n + \liminf b_n$

   (b) $\liminf(a_n + b_n)$

   (c) $\liminf a_n + \limsup b_n$

   (d) $\limsup(a_n + b_n)$

   (e) $\limsup a_n + \limsup b_n$

   (f) $\liminf a_n b_n$

   (g) $\limsup a_n b_n$