Definitions

Lines and Segments

Given two points A and B there is a unique line $\overleftrightarrow{AB}$ containing them. We can identify A with the number 0 and B with any positive real number. This means that each line must have infinitely many points.

The ***distance*** between any two points P and Q on $\overleftrightarrow{AB}$ is defined to be the absolute value of the difference between the two points on the real line associated with P and Q:

$$d(P,Q) = |x_P - x_Q|$$

We will denote this distance by $d(P,Q) \equiv PQ$.

The point C lies ***between*** A and B if A, B and C are distinct points on the same line and $AC + CB = AB$. We will use the notation $A * B * C$ to mean that C lies between A and B.

Given two points A and B, the line ***segment*** $\overline{AB}$ consists of the points A and B and all points that lie between A and B.

$$\overline{AB} = \{A, B\} \cup \{X \in \overleftrightarrow{AB} \mid A * X * B\}.$$

A and B are called the ***endpoints*** and all other points are called ***interior*** points. Note that AB will denote the ***length*** of the segment.

A ***figure*** in the plane is a set of points in the plane. A figure is ***convex*** if it contains all of the interior points of all line segments joining any two points of the figure. A figure that is not convex is called ***concave***.

For two distinct points $A \neq B$, the ***ray*** $\overrightarrow{AB}$ is the set of points consisting of the segment $\overline{AB}$ together with the set of points D so that B lies between A and D.

$$\overrightarrow{AB} = \overline{AB} \cup \{X \in \overleftrightarrow{AB} \mid A * B * X\}.$$

Two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are ***opposite*** if $A * B * C$.

Two rays emanating from the same point form two ***angles*** with the common point called the vertex. The rays form the ***sides*** of the angles. If the two rays coincide, then we have one angle of measure 0° and another angle of measure 360°. If the union of the rays is a straight line, then each angle measures 180°. In this case this is called a ***straight angle***. For all others, one of the angles will have a unique measure between 0° and 180°, exclusive. If the two rays are $\overrightarrow{BA}$ and $\overrightarrow{BC}$ we will denote this angle by $\angle ABC$ and its ***measure*** by $m\angle ABC$. The two rays forming the angles divide the set into two disjoint sets, neither of which contains the sides of the angle. The convex region will be called the ***interior*** of the angle and the concave region will be called the ***exterior***.

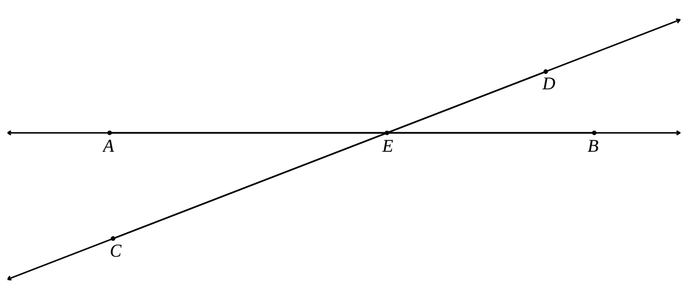
If the measure of an angle lies in (0°,90°) the angle is called ***acute***. If the measure lies in (90°,180°) the angle is called ***obtuse***. If the measure of the angle equals 90°, the angle is called a ***right*** angle.

If the sum of the measures of two angles is 90° they are called ***complementary*** angles. If the sum of the measures is 180° they are called ***supplementary*** angles. Two angles $\angle PQR$ and $\angle RQT$ for a linear pair if P, Q and T are collinear with $P*Q*T$.

Let $A_1, A_2, \ldots, A_n$ be distinct points in the plane so that no three consecutive points are collinear. Suppose that no two of the segments $\overline{A_1A_2}$, $\overline{A_2,A_3}$, ..., $\overline{A_{n-1}A_n}$ and $\overline{A_nA_1}$ share an interior point. The ***n***-gon $A_1A_2\ldots A_n$ is the union of these n segments. The points $A_1, A_2, \ldots, A_n$ are called the ***vertices*** and the segments are the ***sides*** of the polygon.

A polygon divides the plane into two disjoint sets – the ***interior*** of the polygon and the ***exterior*** of the polygon – with no point of the polygon in either set. A polygon is ***convex*** if its interior is a convex region, otherwise we call it ***concave*** or ***non-convex***. A polygon is ***regular*** if all of its sides are congruent and all of its angles are congruent. A 3-gon is called a ***triangle***, a 4-gon is called a ***quadrilateral***, a 5-gon is a ***pentagon***, *etc*. For $n \geq 4$ an interior angle of a polygon may have measure in (180°,360°).

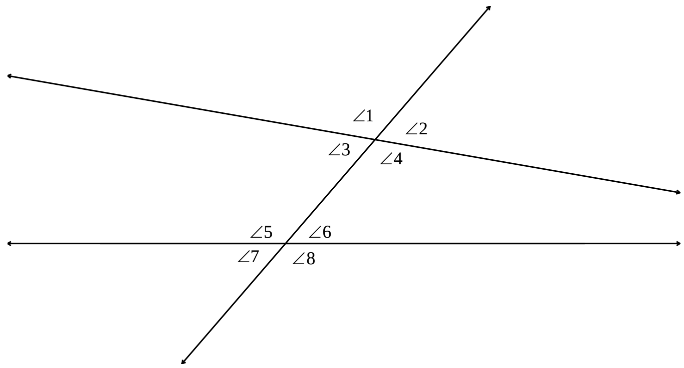
Let P and P' be two plane figures. They are ***similar*** if there exists a positive real number k and an onto function $f : P \to P'$ so that for all $A, B \in P$ $f(A)f(B) = A'B' = k\ AB$. The number k is called the ***coefficient of similarity***. We denote this by $P \sim P'$. If $P \sim P'$ with coefficient of similarity $k = 1$ we say that P and P' are ***congruent***.



Vertical Angles

When two distinct lines intersect they form four angles having a common vertex. The angles in each pair $\{\angle AEC, \angle BED\}$ and $\{\angle CEB, \angle DEA\}$ are called ***vertical*** angles.

Two lines are ***parallel*** if they coincide or they do not intersect. We will indicate that $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel by the notation $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. When two distinct lines , l and m, not necessarily parallel, are intersected by a third line t, called a transversal, then we refer to certain pairs of angles as follows:



The angles in each pair {1,5}, {2,6}, {3,7} and {4,8} are called ***corresponding*** angles.

The angles in each pair {3,6}and {4,5} are called ***alternate interior*** angles.

The angles in each pair {1,8} and {2,7} are called ***alternate exterior*** angles.

The angles in each pair {3,5} and {4,6} are called ***same side interior*** angles.

Two intersecting lines l and m are ***perpendicular*** if the measure of each angle formed by the lines is 90°. If $P \notin l$ there is a unique line through P which is perpendicular to l. Let the point of intersection of this line with l be Q. Q is the ***foot*** of P in l and the length of $\overline{PQ}$ is the ***distance*** from P to l, denoted $d(P,l)$.