## MATH 341 — FALL 2011 ASSIGNMENT 4

## September 21, 2011

Geometry for College Students by Martin Isaacs, page 23:

- 1E.1 Draw two medians of a triangle. This subdivides the interior of the triangle into four pieces: three triangles and a quadrilateral. Show that two of the three small triangles have equal areas and that the area of the third is equal to that of the quadrilateral.
- 1E.2 An arbitrary point *P* is chosen on the base *BC* of an isosceles  $\triangle ABC$  and perpendiculars *PU* and *PV* are drawn from *P* to the other two sides of the triangle. (It may be that *U* or *V* lies on an extension of *AB* or *AC* and not on the actual side of the triangle. This can happen, for instance, if  $\angle A$  is obtuse and point *P* is very near *B* or *C*.) Show that the sum PU + PV of the lengths of the two perpendiculars is constant as *P* move along *BC*. In other words, this quantity is independent of the choice of *P*.
- 1E.3 Since a triangle is determined by angle-side-angle, there should be a formula for  $K_{ABC}$  expressed in terms of *a* and  $\angle B$  and  $\angle C$ . Show that:

$$K_{ABC} = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin(B+C)}.$$