

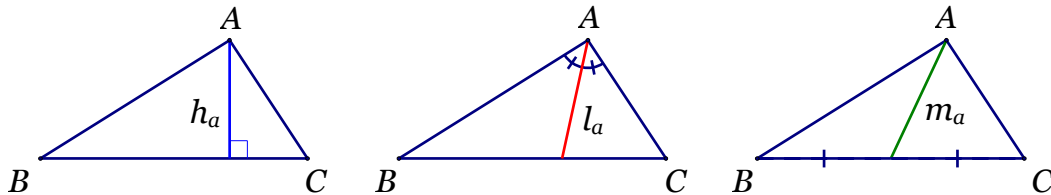
**MATH 341 — FALL 2011**  
**ASSIGNMENT 5**  
**Due Sept 30, 2011**

September 13, 2011

*Geometry for College Students* by Martin Isaacs, pages 32-33:

- 1F.3 Let  $P$  be a point exterior to a circle centered at point  $O$  and draw the two tangents to the circle from  $P$ . Let  $S$  and  $T$  be the two points of tangency. Show that  $OP$  bisects  $\angle SPT$  and  $PS \cong PT$ .
- 1F.5 In  $\triangle ABC$  prove that  $\angle A$  is a right angle if and only if the length of the median from  $A$  to  $BC$  is exactly half the length of side  $BC$ .

For the remainder of the semester we will use the following notation. For  $\triangle ABC$  we let  $a$ ,  $b$ , and  $c$  denote the lengths of the sides opposite  $A$ ,  $B$  and  $C$  respectively, i.e.  $a = BC$ ,  $b = AC$ , and  $c = AB$ . Define the *semiperimeter*  $s$  to be  $s = \frac{a+b+c}{2}$ . Let  $h_a$ ,  $h_b$ , and  $h_c$  denote the lengths of the altitudes from  $A$ ,  $B$ , and  $C$ , respectively. Let  $l_a$ ,  $l_b$ , and  $l_c$  denote the lengths of the segments of the angle bisectors from  $A$ ,  $B$ , and  $C$ , respectively, from the vertex to the opposite side. Let  $m_a$ ,  $m_b$ , and  $m_c$  denote the lengths of the medians from  $A$ ,  $B$ , and  $C$ , respectively.



The above figure shows  $h_a$ ,  $l_a$ , and  $m_a$ .

3. Consider  $\triangle ABC$  with  $a = 8$ ,  $b = 3$  and  $c = 6$ . Let  $AD$  be the altitude at  $A$ ,  $AF$  the bisector at  $A$ , and  $AM$  be the median at  $A$ .
- Find  $CD$ ,  $CF$ , and  $CM$ .
  - Find  $h_a$ ,  $l_a$ , and  $m_a$ .
4. Given any triangle  $\triangle ABC$  show that  $h_a$  can be expressed by the following formula without using Stewart's Theorem

$$h_a = \frac{2}{a} \sqrt{s(s-a)(s-b)(s-c)}$$