# MATH 341 - FALL 2011 ASSIGNMENT 9 

## Due October 31, 2011

9.1 [Geometry for College Students by Martin Isaacs, page 72, 2D.2] A quadrilateral with side lengths $1,1,1$, and $d$ is inscribed in a circle. Find a formula for the radius $R$ of the circle in terms of $d$. Check your formula by computing $R$ directly when $d=1$. HINT: Prove that the diagonals of the quadrilateral must be equal.

9.2 Find the area of a cyclic quadrilateral whose sides have lengths 9, 10, 10. and 21.
9.3 Find the area of a cyclic quadrilateral whose sides have lengths $7,15,20$. and 24 .
9.4 Use GeoGebra do complete this problem. Take an arbitrary convex quadrilateral, $\square A B C D$. Using the diagonals find the orthocenter of each of the four triangles. Connect the orthocenters in order giving you another convex quadrilateral, call it $\square P Q R S$.
(a) Find the area of $\square A B C D$.
(b) Find the area of $\square P Q R S$.
(c) Find the ratio $K_{A B C D}: K_{P Q R S}$.
9.5 Again using GeoGebra take an arbitrary convex quadrilateral, $\triangle A B C D$. Use the diagonals and construct the quadrilaterals as above for the

- circumcenter and
- incenter.

For each of these,
(a) Find the area of $\square A B C D$.
(b) Find the area of $\square P Q R S$.
(c) Find the ratio $K_{A B C D}: K_{P Q R S}$.

Are any of these ratios constant?
9.6 To what familiar result does Ptolemy's theorem lead when the cyclic quadrilateral is a rectangle? Prove your result.
9.7 [Challenge Question] Using GeoGebra start with a triangle $\triangle A B C$ and construct a point $D$ so that the convex quadrilateral $\square A B C D$ is bicentric.
Challenge: Find a technique that will work for any triangle and prove that it works.

