MATH 4181 001 Fall 1999

Problem Set 4

1. (Exercise 4, page 74) Let $(X, \mathscr{T})$ be a topological space. Prove that $\emptyset$ and X are closed sets, that a finite union of closed sets is a closed set, and that an arbitrary intersection of closed sets is a closed set.

2. (Exercise 6, page 75) Prove that in a discrete topological space, each subset is simultaneously open and closed.

3. Show that a topological space $(X, \mathscr{T})$ is discrete if and only if each set consisting of only one point is open.

4. Let X be a set and let $\mathscr{T}'$ the finite complement topology for X.
   (a) Show that $(X, \mathscr{T}')$ is discrete if and only if X is a finite set.
   (b) Show that if A is an infinite subset of X, then every point of X is a limit point of A.

5. Let X be a set. The *countable complement topology*, or *co-countable topology*, $\mathscr{T}''$ for X consists of X, $\emptyset$ and all subsets O of X for which $X \setminus O$ is a countable set.
   (a) Show that $\mathscr{T}''$ is a topology on X.
   (b) For the space $(X, \mathscr{T}'')$, show that a countable set A of X has a derived set $A' = \emptyset$ and that an uncountable set B has $B' = X$.
   (c) Show that the intersection of any countable family of members of $\mathscr{T}''$ is a member of $\mathscr{T}''$.

6. Let $X = \{a, b\}$ be a two-element set and let $\mathscr{T} = \{\emptyset, \{a\}, \{a, b\}\}$. Show that $\mathscr{T}$ is a topology on X and identify the limit points of each subset of X. (This space is called *Sierpenski space.*)

7. How many different topologies are there for a set with three members?