September 23, 2009

Name: ______________________________
Section: ______

MA 201: Exam I Practice

Please read the following carefully.

- There are 10 questions on this exam and there are 10 points possible for each question. The exam is worth 100 points total.
- You may use a simple calculator but you may not use a cellphone or calculator which stores notes.
- For any question which asks you to explain something you must write in complete English sentences. You can lose points for incomplete or incomprehensible explanations.
- For any computation problem you must show all work. You will lose points if it is not made clear how you arrive at an answer.
- Follow all instructions carefully. If a problem says to use a particular method, you *must* use that method. No points can be awarded if you fail to use the specified method.
- Relax and don't spend too much time on any one problem! Good luck.

Question	Possible	Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

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1. (a) Let A and B be sets. Give a verbal and a mathematical definition for $A \cup B$, $\bar{A}$, and $\emptyset$.

> **Solution:** $A \cup B$ is the set of all elements in A or in B. In symbols
>
> $$A \cup B = \{x \in U | x \in A \text{ or } x \in B\}.$$
>
> $\bar{A}$ is the set of elements in the universe but not in A. In symbols,
>
> $$\bar{A} = \{x \in U | x \text{ is not in } A\}$$
>
> $\emptyset$ is the set of no elements. In listing notation $\emptyset = \{\}$.

   (b) Let $U = \{x | x$ is a letter in the English alphabet or x is a whole number less than 5$\}$. Let A be the set of letters in the alphabet, $B = \{x \in U | 0 < x \leq 3\}$, and $C = \{a, b, c, 1, 4\}$. Find $\bar{A}$, $A \cap C$, and $B \cup C$.

> **Solution:**
>
> $$\bar{A} = \{0, 1, 2, 3, 4\}$$
> $$A \cap C = \{a, b, c\}$$
> $$(B \cup C = \{a, b, c, 1, 2, 3, 4\}$$

2. (a) Your classmate claims that $\{0\} = \emptyset$. Explain why she is incorrect and how you would address her misunderstanding.

> **Solution:** The empty set, $\emptyset$, is the set which contains no elements. The set $\{0\}$ is the set which contains the element 0. Since the two sets have different numbers of elements they cannot possibly be equal. One way to explain the difference is to think of sets as bags. Then the $\emptyset$ is an empty bag but the set $\{0\}$ is the set which contains the number 0.

   (b) If $A \subset B$ and $B \subset C$ is $A \subset C$? Use Venn diagrams to explain your answer. Give a practical example of sets A, B, and C which demonstrates this idea.

3. (a) Explain what it means for two sets to be equivalent.

> **Solution:** Two sets are equivalent if they correspond one-to-one. That is, $A \sim B$ if every element of A can be paired with a unique element of B so that every element of B has exactly one partner from A.

   (b) Explain what it means for two sets to be equal.

Solution: Two sets are equal when they have the same elements.

(c) Show that the set of whole numbers is equivalent to the set of natural numbers. Write in complete sentences and clearly demonstrate your one-to-one correspondence.

Solution: Let W be the set of whole numbers and N be the set of natural numbers. Then we can define the one to one correspondence $n = w + 1$ for every $w \in W$. This shows that $W \sim N$.

(d) Explain why the set of whole numbers and the set of natural numbers are not equal.

Solution: The sets are not equal because the whole numbers contain the element 0 but the natural numbers do not.

4. There are 100 students in the sixth grade and there are three after school clubs: art, music, and sports. Suppose 60 students are in the art club, 30 are in the music club, and 30 are in the sports club. Suppose also that 20 students are in both art and music, 15 are in art and sports, and 10 are in both music and sports. There are 5 students in all three clubs. How many students are in more than one club and how many are in no clubs? You must show your work.

Solution: Diagram is omitted...
There are 35 students in more than one club.
There are 20 students in none of the three clubs.

5. State Polya's four principles and explain each principle with 1-2 sentences.

Solution: This is in your text-book.

6. If a and b are whole numbers both divisible by c prove that $a - b$ is also divisible by c.

Solution: If c divides a, then $a = x * c$ for some number x. If c divides b, then $b = y * c$ for some number y. This means that $a - b = xc - yc = (x - y) * c$ which means that $a - b = z * c$ for some number z. This means that $a - b$ is divisible by c.

7. Solve the following problem using the four Polya principles. You should indicate where and how you are applying each step of the principles. Write up your answer as if you were presenting it to a classroom. That is, your write up should be brief (one or two sentences per step) and well organized.

Problem. Find the n^{th} term of the sequence: $2, 6, 10, 14, 18, ...$

Solution: The explanation of the Polya principles is omitted. See the book for an example on how to do this.

The n^{th} term of the sequence is $(2n-1)*2$. I noticed this because the first term is 2, which is $1*2$. The second term is $3*2$. The third term is $5*2$. So, I needed to have the n^{th} odd number multiplied by 2 at the n^{th} step. The n^{th} odd number is $2n-1$. So, the n^{th} term is $(2n-1)*2$.

8. Solve the following problem using the four Polya principles. You should indicate where and how you are applying each step of the principles. Write up your answer as if you were presenting it to a classroom. That is, your write up should be brief (one or two sentences per step) and well organized.

   Problem. Suppose that you can paint a house in 3 hours and your friend can paint a house in 4 hours. How long will it take to paint a house if you work together?

Solution: If I can paint a house in 3 hours, then I can paint $\frac{1}{3}$ of a house in 1 hour. If you can paint a house in 4 hours, then you can paint $\frac{1}{4}$ of a house in 1 hour.

If we work for H hours to paint the whole house, then I will paint $\frac{H}{3}$ of the house, and you will paint $\frac{H}{4}$ of the house.

If we add up how much we each paint, then we want to paint 1 total house. So, we should have that $\frac{H}{3}+\frac{H}{4}=1$. Multiply both sides by 3, and we see that $H+\frac{3H}{4}=3$. Multiply both sides by 4, and we see that $4H+3H=12$. This means that $7H=12$. This means that $H=\frac{12}{7}$.

Working together we will paint the house in $\frac{12}{7}$ hours.

9. Prove that the product of an even number and an even number is always an even number.

Solution: Let a be an even number. Let b be an even number. Then $a=2x$ and $b=2y$ for some numbers x and y. If we multiply a and b, then we get $a*b=2x*2y=4xy$. But $a*b=4xy=2*(2xy)$. This means that $a*b$ is 2 times a number. This means that $a*b$ is even.

10. (a) State the pigeonhole principle.
    (b) If there are 250,000 people in Lexington and any person can have at most 200,000 hairs on their head, explain why you can conclude that a least two people will have the same number of hairs on their head.

Solution: (a) The pigeonhole principle states that if we have m pigeons and n holes, and $m > n$ (we have more pigeons than holes), then there must be at least one hole with 2 pigeons in it.

(b) Think of the people as pigeons. Think of the number of hairs on a head as the holes. Because 250,000>200,000, by the pigeonhole principle, there must be at least two people with the same number of hairs on their head.

11. Answer the following questions regarding mathematical reasoning with a 1-2 sentence explanation for each question.
    (a) Define inductive and deductive reasoning.

    (b) We solved many types of problems using the pattern recognition strategy. What type of mathematical reasoning do you use in this strategy? Explain.

    (c) Suppose that you know $a \to b$. If b is false what can you conclude?

    (d) Suppose that you know $a \to b$. If a is true what can you conclude?

    (e) Suppose that $a \to b$ is true and that b is true. Is a true?

    (f) If $a \to b$ is true and a is true then is b true?

Solution: (a) This is in your book.
(b) Inductive reasoning. This is exactly what inductive reasoning is. We look for a pattern, and we try to find more examples of how it is true. If we find something that disagrees with this pattern, it is a counterexample, and we decide we were wrong. If we don't find a counterexample, then we make a conjecture and we try to prove it is true in general.
(c) If b is false, then a is false by indirect reasoning.
(d) If a is true, then we conclude b is true by direct reasoning.
(e) No. This is a form of invalid reasoning. This is the converse of the statement $a \to b$ and we do not know if $b \to a$.
(f) Yes. This is exactly the same as (d). This is direct reasoning.