September 28, 2010

Name: Answer Key
Section: 001 & 003

MA 201: Exam I

Please read the following carefully.

- There are 10 questions on this exam and there are 10 points possible for each question. The exam is worth 100 points total.
- You may use a simple calculator but you may not use a cellphone or calculator which stores notes.
- For any question which asks you to explain something you must write in complete English sentences. You can lose points for incomplete or incomprehensible explanations.
- For any computation problem you must show all work. You will lose points if it is not made clear how you arrive at an answer.
- Follow all instructions carefully. If a problem says to use a particular method, you *must* use that method. No points can be awarded if you fail to use the specified method.
- Relax and don't spend too much time on any one problem! Good luck.

Question	Possible	Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (a) Let A and B be sets. (4pts)

- Define $A \cup B$.

The set of all elements in A or in B.

- Define $\bar{A}$.

The set of all elements in the universe but not in A.

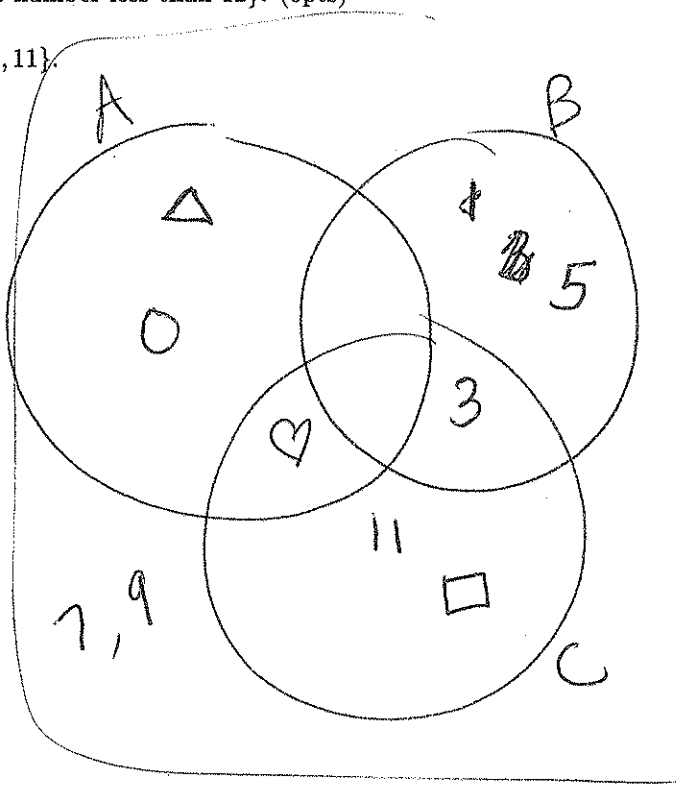
- Define $\emptyset$.

The set with no elements.

- Describe what is meant by the symbol $n(A)$.

The cardinality or number of elements in A.

(b) Let $U = \{x | x$ is $\bigcirc, \heartsuit, \triangle, \square,$ or x is an odd whole number less than $12\}$. (6pts)
Let $A = \{x | x \in U$ and x is $\heartsuit, \triangle, \bigcirc\}$,
$B = \{x | x \in U$ and $0 < x \leq 5\}$, and $C = \{\heartsuit, \square, 3, 11\}$.



- Find $\bar{A}$.

1, 3, 5, 7, 9, 11, $\square$

- Find $A \cap B \cap C$.

$\emptyset$

- Find $A \cap C$.

$\heartsuit$

- Find $A \cup C$.

$\heartsuit, \triangle, 0, \square, 3, 11$

2. (a) Your classmate claims that $n(\emptyset) = 0$. Is she correct or incorrect? Explain. (3pts)

Correct!
There is nothing in the empty set, so there are zero things in the empty set, so

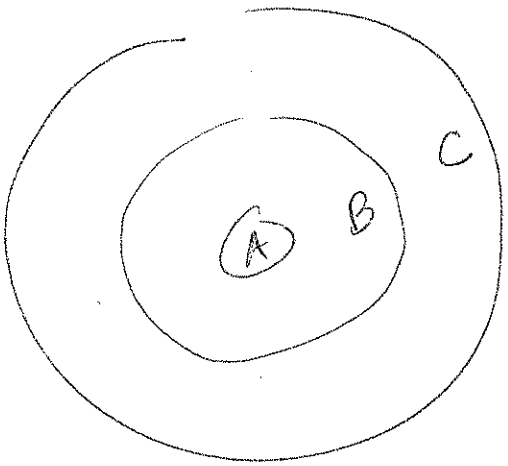
$n(\emptyset) = 0$.

(b) Your friend claims that $n(\{0\}) = 0$. Is she correct or incorrect? Explain. (3pts)

No, she is Incorrect!

There is one thing in $\{0\}$. That thing is 0.
So $n(\{0\}) = 1 \neq 0$.

(c) If $A \subset B$ and $B \subset C$ is $A \subset C$? Use Venn diagrams to explain your answer. Give an example of sets A, B, and C which demonstrates this idea. (4pts)



$A = \{0\}$
$B = \{0, 1, 2\}$
$C = \{0, 1, 2, 3, 4, 5\}$.

3. (a) What does it means for two sets to be equivalent? (1pt)

It means there is a one to one correspondence between the element. Each element can be mapped to exactly one other element.

(b) What does it means for two sets to be equal? (1pt)

They have exactly the same elements.

(c) Let $A = \{1, 3, 5, 7, 9, ...\}$. Let $B = \{x | x \text{ is an odd natural number}\}$. Let C be the set of all even natural numbers. (8pts)

- Does $A = B$? Explain.

Yes. $B = \{1, 3, 5, 7, 9, ...\}$. A & B have exactly the same elements.

- Is $A \sim B$? Explain and clearly demonstrate your one-to-one correspondence if there is one.

Yes. They are equal. Just map each element to itself. $n \longmapsto n$.

- Is $A = C$? Explain.

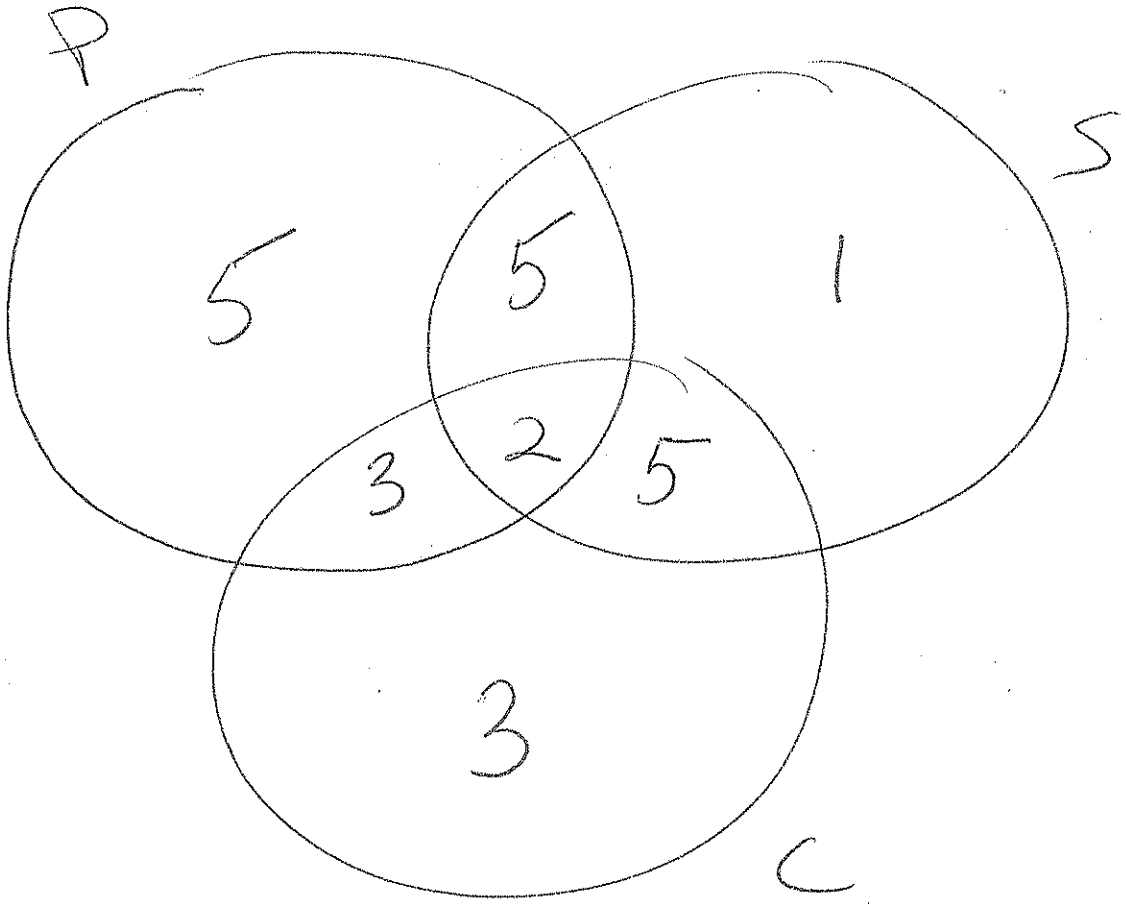
No. Evens are not the same thing as odds. For example, $2 \in C$ but $2 \notin A$.

- Is $A \sim C$? Explain, and clearly demonstrate your one-to-one correspondence if there is one.

Yes.

A	C
$1 \longmapsto$	2
$3 \longmapsto$	4
$5 \longmapsto$	6
$\vdots$	$\vdots$
$n \longmapsto$	$n+1$.

4. There are 25 students in our class. After taking a poll, I discovered that of the three foods, pizza, salad and cookies: 15 students like pizza, 13 students like salad, 13 students like cookies. I also know that 7 students like both pizza and salad, 7 like both salad and cookies, and 5 like both pizza and cookies. There are 2 students who like all three. How many students like exactly one of the three items? How many like none of the three items? How many students like cookies, but do not like pizza or salad? You must show your work. (10pts)



$5+1+3=9$ like exactly one.

$25 - 24 = 1$ like none of the three.

3 like cookies but not pizza or salad.

5. (a) It takes Julie 5 hours to finish her math homework, but only takes Rebecca 3 hours. If they work together, how long will it take them to finish their math homework? (7pts)

Julie finishes $\frac{1}{5}$ of h/w in 1 hr.

Rebecca finishes $\frac{1}{3}$ of h/w in 1 hr.

In H hours Julie finishes $\frac{H}{5}$ and Rebecca finishes $\frac{H}{3}$.

Together they want to finish 1 assgt.

So we want

$$\frac{H}{5} + \frac{H}{3} = 1 \Rightarrow \frac{3H}{5} + H = 3$$

$$\Rightarrow 3H + 5H = 15 \Rightarrow 8H = 15 \Rightarrow H = \frac{15}{8} \text{ hours}$$

(b) Your student's answer to the question above was 4 hours. Briefly explain why they are incorrect, and what they might have done to get this answer. (3pts)

They are incorrect because they took the average. If Rebecca can finish in 3 hours, it shouldn't take her longer to work with someone.

6. (a) Write the first 6 terms of the Fibonacci sequence. (2pts)

1, 1, 2, 3, 5, 8

(b) Use Gauss's Rule to find the sum of the first 102 natural numbers. (2pts)
(In other words, what is $1+2+3+4+\cdots+100+101+102$?)

$$\frac{(102)(103)}{2} = 51(103) = 5253$$

103
51
103
5253

(c) The sixth row of Pascal's Triangle is 1 6 15 20 15 6 1. What is the next row? (2pts)

1, 7, 21, 35, 21, 7, 1

(d) Finish the following sequences by filling in the next 4 numbers (4pts)

- 64, 32, 16, 8, ... 4, 2, 1, 1/2

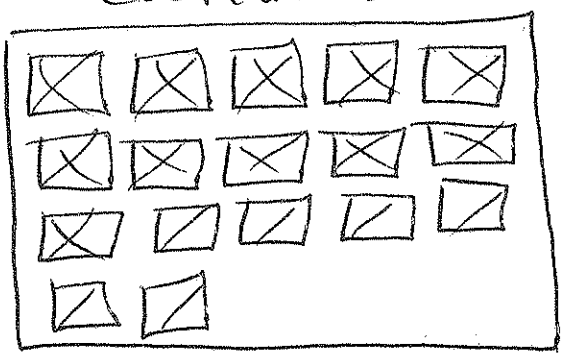
- 1, 4, 9, 16, 25, ... 36, 49, 64, 81

7. (a) Solve the following problem using the draw a diagram strategy. You should indicate where and how you are applying each step of Polya's Principles. Your answer should only be 1 sentence per step, and well organized. (7pts)
Problem:
John has 28 CDs, but he was irresponsible and lost some of his CD cases. He only has 17 of the cases now. Assuming no case contains more than 2 CDs and every CD is in a case, find out how many CD cases contain 2 CDs.

(1) Understand the Problem: I know there are more CDs than cases, so some will need to contain more than 1.

(2) Make a Plan: I will draw a diagram.

(3)



put 2 lines for 2 cd's and label until you get to 28. There are (11) with 2 CD's inside

(4) Look Back: This strategy worked well, and was very visual.

(b) State the Pigeon-hole Principle. (1pt)

If there are m pigeons and n holes (m>n) then some hole must have more than one pigeon.

(c) Explain why if every CD must be in a case, that there must be at least one case that contains two CDs. (You do not need to describe Polya's Principles on this section.) (2pts)

Because CDs are like pigeons, cases are like holes. There are more CDs than cases, so use the pigeonhole principle.

8. (a) Define "manipulative" and give an example. (2pts)

An object used to demonstrate mathematical concepts.

(number line)

(b) List the two models we learned for whole number addition. (2pts)

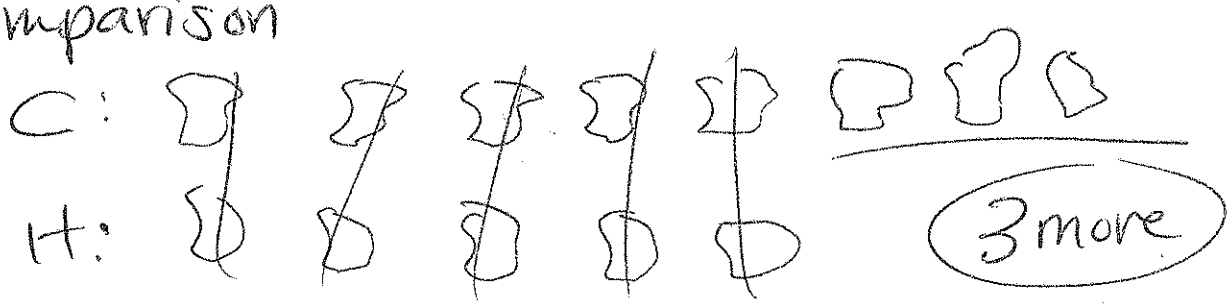
Set model

Measurement / # line model

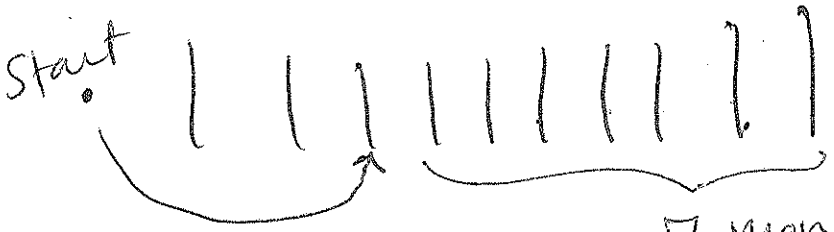
(c) On the following problems, first identify which subtraction model best fits, then use that model to solve the problem. (6pts)

- Claire has 8 silly bands, but Hannah has 5. How many more silly bands does Claire have than Hannah?

Comparison



- There are ten monkey bars. You have already made it to the third bar. How many more bars do you have to travel until you reach the end?



(# line model or missing addend)

9. Use Inductive Reasoning by following the steps below:

(a) Briefly explain the process of inductive reasoning. (4pts)

Look for a pattern. Try more examples. Look for a counter example & deduce you are wrong OR: Make a conjecture and prove it.

Consider function $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$.

I tested the values $f(1), f(2), f(3), f(4)$ by plugging in $1, 2, 3, 4$ for x and found that:

$$f(1) = 0$$
$$f(2) = 0$$
$$f(3) = 0$$
$$f(4) = 0.$$

(b) Make a conjecture about what the value of $f(x)$ is for any whole number x. (2pts)

$f(x) = 0$ for any whole # x.

(c) Is 0 a whole number? (1pt)

Yes.

(d) What is $f(0)$? (1pt)

$f(0) =$ 24

(e) Was your conjecture in (b) correct? Explain. (2pts)

No! Because $f(0)$ is a counterexample.

10. If n is a multiple of 3, then n^2 is a multiple of 9. This statement is true. (You do not need to prove this.)

(a) Identify the "p" statement. (1pt)

"n is a multiple of 3"

(b) Identify the "q" statement. (1pt)

"n^2 is a multiple of 9"

(c) If we know that n^2 is **not** a multiple of 9, do we know that n is **not** a multiple of 3? (2pts)

Yes.

(d) If yes, what type of deductive reasoning did you use in (c)? If no, explain. (1pt)

Indirect reasoning.

(e) Notice that $1386 = 462 * 3$, which means that 1386 is a multiple of 3. Do we know that 1386^2 is a multiple of 9? (2pts)

Yes.

(f) If yes, what type of deductive reasoning did you use in (e)? If no, explain. (1pt)

Direct reasoning.

(g) If we know that n^2 **is** a multiple of 9, do we know that n **is** a multiple of 3? (1pt)

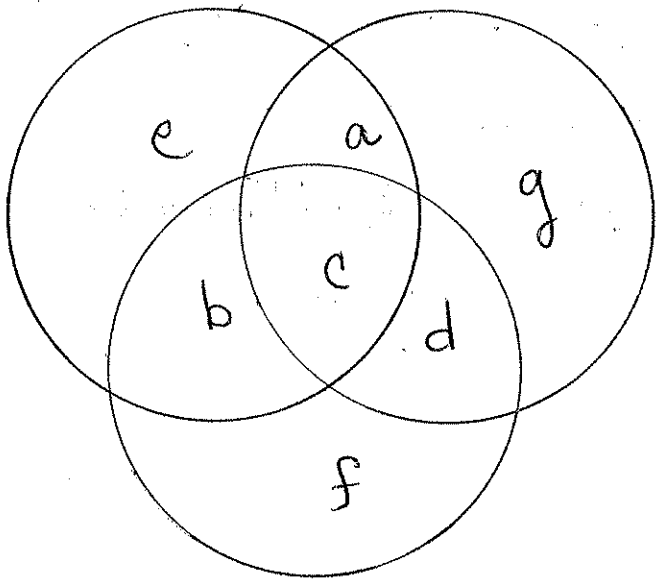
No!

(h) If yes, what type of deductive reasoning did you use in (g)? If no, explain. (1pt)

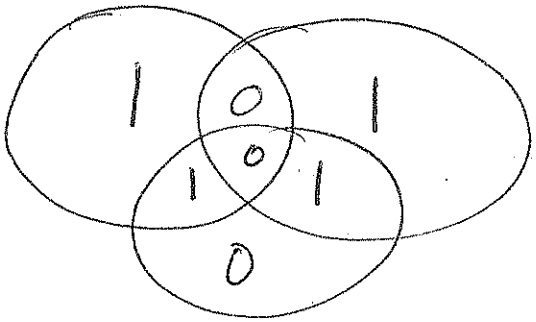
This is invalid reasoning. We don't know that because $p \to q$ we have $q \to p$.

Extra Credit

11. Recall that a seven-bit word in the Hamming Code can be represented by a Venn diagram. Also, the ordered labeling of a code $\{a, b, c, d, e, f, g\}$ is shown in the figure below. Remember that each circle must contain exactly two 0s and two 1s. Use this to answer the following problems:



(a) Extend the code $\{a, b, c, d\} = \{0, 1, 0, 1\}$ to a seven-bit Hamming Code by filling in the Venn diagram corresponding to the figure above. (2pts)



The 7-bit word is $\{0, 1, 0, 1, 1, 0, 1\}$

(b) At most one bit is in error in the Hamming code word $\{a, b, c, d, e, f, g\} = \{0, 0, 1, 1, 1, 1, 1\}$. Determine whether this word is correct, or if it is incorrect, correct the code. (3pts)

Corrected Code: $\{0, 0, 1, 0, 1, 1, 1\}$