Name: Answer Key

Section:001

December 2, 2010

MA 201-001: Exam III

Please read the following carefully.

- There are 10 questions on this exam and there are 10 points possible for each question. The exam is worth 100 points total.
- You may **not** use a calculator.
- For any question which asks you to explain something you must write in complete English sentences. You can lose points for incomplete or incomprehensible explanations.
- For any computation problem you must show all work. You will lose points if it is not made clear how you arrive at an answer.
- Follow all instructions carefully. If a problem says to use a particular method, you *must* use that method. No points can be awarded if you fail to use the specified method.
- Relax and don't spend too much time on any one problem! Good luck.

Question	Possible	Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. **§3.1**

(a) Write 63, 491 in expanded notation. (4 pts)

$6 \times 10{,}000 + 3 \times 1{,}000 + 4 \times 100 + 9 \times 10 + 1 \times 1$

(b) Add 343 + 277 like an Egyptian would, using the symbols listed below. (3 pts)

= 9

= ∩

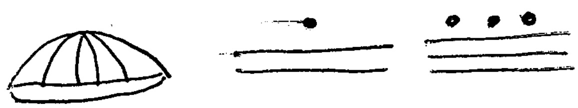
999∩∩∩∩|||

+ 99∩∩∩∩∩∩∩|||||||

999999∩∩

620

(c) What Mayan number does the following represent in the system we use today? (3 pts)



$0 \times 18 \times 20 \quad 11 \times 20 \quad 18 \times 1$

$0 + 220 + 18 = 238$

TABLE 3.1 Egyptian Symbols for Powers of 10

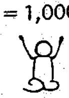
Power of 10	$10^0 = 1$	$10^1 = 10$	$10^2 = 100$	$10^3 = 1000$	$10^4 = 10{,}000$	$10^5 = 100{,}000$	$10^6 = 1{,}000{,}000$
Egyptian Symbol	|	∩	9				
Description	a staff	a yoke	a scroll	a lotus flower	a pointing finger	a fish	an amazed person

TABLE 3.3 Mayan Numerals for 0 through 19

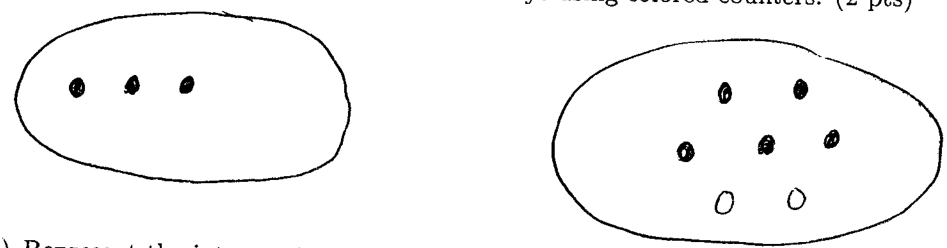
Mayan Symbol	Modern Equivalent	Mayan Symbol	Modern Equivalent
	0		10
	1		11
	2		12
	3		13
	4		14
	5		15
	6		16
	7		17
	8		18
	9		19

2. §5.1

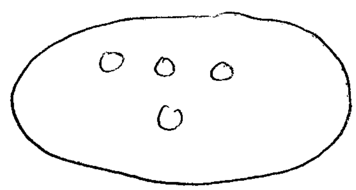
For the following question let • represent a positive integer, and ∘ represent a negative integer, as in colored counters.

(a) Represent the integer 3 in two different ways using colored counters. (2 pts)

one way

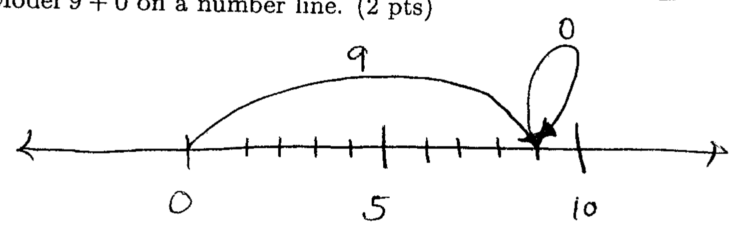


(b) Represent the integer -4 using colored counters. (1 pt)



(c) Demonstrate using colored counters that $4 + (-4) = 0$. (2 pts)

(d) Model $9 + 0$ on a number line. (2 pts)



(e) Define absolute value. (1 pt)

The distance a number is from zero.

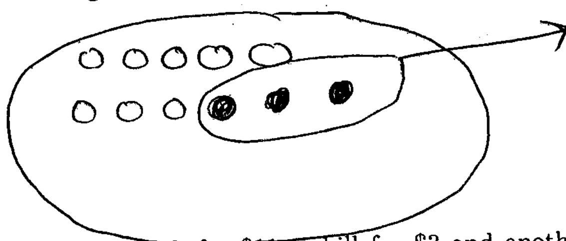
(f) Fill in the box with $<, =, >$. (2 pts)

(a) $|7| \square |-3|$ $>$

(b) $|-4| \square |4|$ $=$

3. §5.2 For the following questions let • represent a positive integer, and ∘ represent a negative integer, as in colored counters.

(a) Draw a diagram using colored counters for $-5-(3)$ and solve. (2pts)



$= -8$

(b) The mailman brings a check for $11, a bill for $3 and another bill for $4. Are you richer or poorer? By how much? (Be sure to write the correct equation before you solve.) (2pts)

$11+(-3)+(-4) = 11-7 = 4$

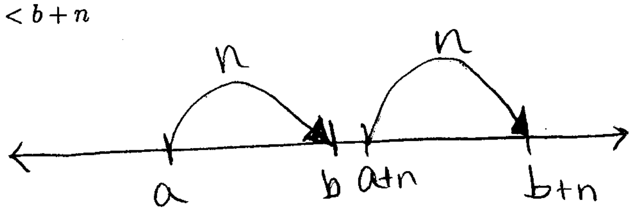
$4 richer.

(c) The mailman brings you a check for $11 and takes away a bill for $6. Are you richer or poorer? By how much? (Be sure to write the correct equation before you solve.) (2pts)

$11-(-6) = 11+6 =$ $17 richer.

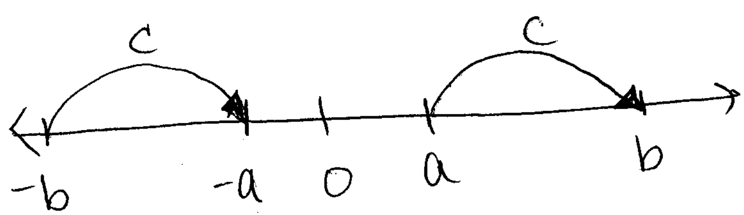
(d) Use a number line diagram to show that if $a, b, n \in \mathbb{Z}$ and $a < b$ that the following are true: (4pts)

(a) $a+n < b+n$



(doesn't matter where zero is)

(b) $-b < -a$



If c is the distance from a to b it must stay the same because -b is b reflected across the 0.

4. §5.3

(a) Fill in the boxes with either a + or a − sign. (We are assuming that m, n are both positive integers. When you see $-m$, that will represent a negative integer, and $+m$ represents a positive integer.) (2pts)

(a) $(+m) \cdot (+n) = \square mn$ +

(b) $(-m) \cdot (+n) = \square mn$ −

(c) $(+m) \cdot (-n) = \square mn$ −

(d) $(-m) \cdot (-n) = \square mn$ +

(e) The letter carrier takes away 4 bills for \$16 each. Are you richer or poorer? By how much? (Be sure to write the correct equation before you solve.) (2pts)

$-4 \times (-16) = 64$

\$64 richer

(f) Complete the following divisions: (2pts)

(a) $12 \div (-4) =$ -3

(b) $-12 \div (-4) =$ 3

(c) Write a mail-time story to model the following expression, and decide if you are richer or poorer, and by how much: (4 pts)

$$2 \times (-4) - 3 \times (5) =$$

The mailman brings 2 bills for \$4 and takes 3 checks for \$5.

$-8 - 15 = -23$

\$23 poorer.

5. **§5.4**

(a) Compute $8 +_{12} 5 +_{12} 3 =$ (2pts) 4

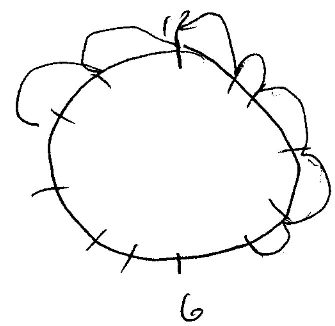
$13+3 = 16$

$12\overline{)16}$ 1, 12, 4

(b) Compute $5 -_{12} 7 =$ (2pts)

$5 -_{12} 7 = 5+5 = 10$

$12-7=5$



(c) Compute $5 \times_{12} 5 \times_{12} 5 =$ (2pts)

$12\overline{)125}$ 10, 12, 05, 5

$= 5$

(d) Compute $8 \div_{12} 5 =$ (2pts) 4

$8\times 5 = 40$

$12\overline{)40}$ 3, 36, 4

$8 \div_{12} 5 = x$

means $8 = 5 \times_{12} x$

(e) Compute $7 \times_{10} 4 =$ (2pts)

$7\times 4 = 28$

$10\overline{)28}$ 2, 20, 8

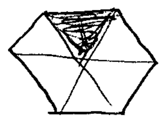
8

6. §6.1 **Modeling Fractions/Equivalent Fractions**
Be sure to identify your units on the following problems.

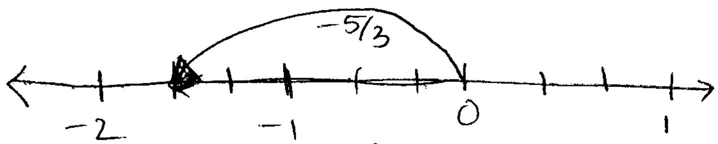
(a) How many different representations are there for the fraction $\frac{1}{2}$? (1pt)

Infinitely many.

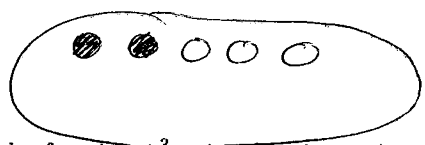
(b) Represent the fraction $\frac{1}{6}$ using the colored region model. (1pt)



(c) Represent the fraction $\frac{-5}{3}$ using the number line model. (1pt)



(d) Represent the fraction $\frac{2}{5}$ using the set model. (1pt)



2 out of 5 circles are shaded. My unit is 5 circles.

(e) Represent the fraction $1\frac{3}{5}$ using fraction strips. (2pts)

(f) State the Cross-Product Property of Equivalent Fractions given that $\frac{a}{b}$ and $\frac{c}{d}$ are fractions. (In other words, how do we know when $\frac{a}{b} = \frac{c}{d}$?) (2pts)

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc.$$

(g) Is $\frac{14}{6} = \frac{17}{8}$? Explain. (2pts) No.

Can see that $14 \times 8 \neq 6 \times 17$ because
$112 \neq 102$ OR:

get Common denom:

$$\frac{14}{6} = \frac{14}{6} \times \frac{4}{4} = \frac{56}{24}$$

$$\frac{17}{8} = \frac{17}{8} \times \frac{3}{3} = \frac{51}{24}$$

and $56 \neq 51$.

7. §6.1 **Simplifying fractions/Common Denominators/Ordering**

(a) Write $\frac{24}{-72}$ in simplest terms. Explain how you know it is in simplest terms. (2 pts)

$$\frac{24}{-72} = \frac{12}{-36} = \frac{6}{-18} = \frac{3}{-9} = \frac{1}{-3} = \frac{-1}{3}$$

This is in simplest terms because the denominator is positive and gcd$(-1,3)=1$.

(b) Write $\frac{294}{84}$ in simplest terms given that $294 = 2 \cdot 3 \cdot 7^2$ and $84 = 2^2 \cdot 3 \cdot 7$. (2pts)

$$\frac{294}{84} = \frac{\cancel{2} \cdot \cancel{3} \cdot 7^{\cancel{2}}}{2^{\cancel{2}} \cdot \cancel{3} \cdot \cancel{7}} = \frac{7}{2}$$

(c) Explain in 2-3 sentences how to get two equivalent fractions but with a common denominator for the fractions $\frac{2}{7}$ and $\frac{5}{21}$. (3pts)

Since 21 is a multiple of 7, we can leave $\frac{5}{21}$ as it is. Then, ask ourselves "what should we multiply 7 by to get 21?" Since we must multiply by a form of 1 we multiply by $\frac{3}{3}$. $\frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$. So, our two fractions are $\frac{5}{21}$ and $\frac{6}{21}$.

(d) Show that $\frac{a}{b} < \frac{c}{d}$ when $ad < bc$. (3pts)

Get a common denominator

$$\frac{a}{b} \times \frac{d}{d} = \frac{ad}{bd}$$

$$\frac{c}{d} \times \frac{b}{b} = \frac{bc}{bd}.$$

Now compare

$$\frac{a}{b} \; \square \; \frac{c}{d}$$

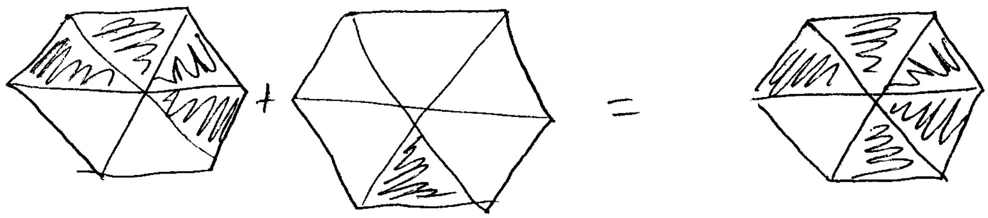
$$\Rightarrow \frac{ad}{bd} \; \square \; \frac{bc}{bd}$$

and because $ad < bc$ we put $<$ everywhere.

So that $a/b \boxed{<} c/d$.

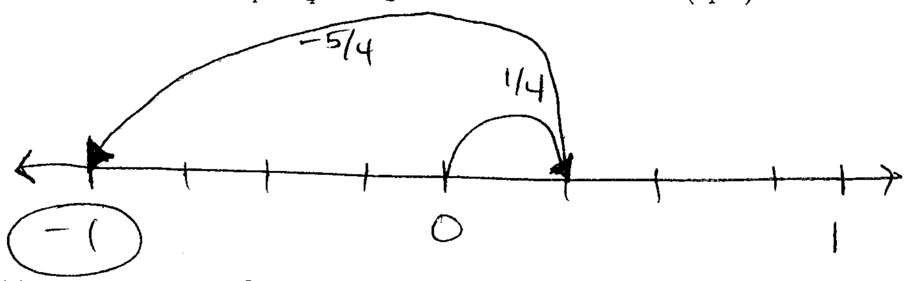
8. §6.2 **Adding and Subtracting Fractions/Mixed Numbers vs. Proper Fractions**

(a) Model and solve $\frac{4}{6} + \frac{1}{6}$ using the shaded region model. (1pt)



$= \frac{5}{6}$

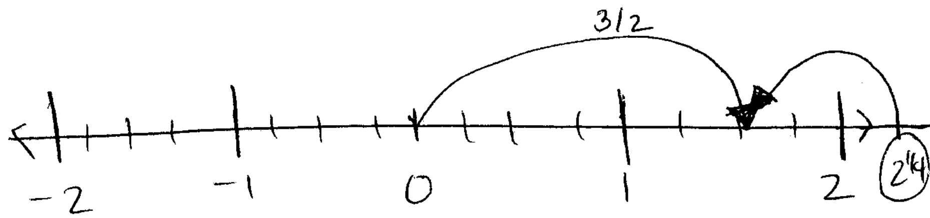
(b) Model and solve $\frac{1}{4} + \frac{-5}{4}$ using the number-line model. (2pts)



head to tail

$= -1$

(c) Model and solve $\frac{3}{2} - \frac{-3}{4}$ using the number-line model.(3pts)



heads together

$2\frac{1}{4}$

(d) Give an improper fraction for $3\frac{8}{9}$. (Show your work.) (2pts)

$$3\frac{8}{9} = \frac{(3 \times 9) + 8}{9} = \frac{27+8}{9} = \frac{35}{9}$$

(e) Compute $2\frac{1}{4} + 5\frac{2}{5}$. (Show your work.) (2pts) $= 2+5+\frac{1}{4}+\frac{2}{5} = 7+\frac{5}{20}+\frac{8}{20} = 7\frac{13}{20}$

OR:

$$\frac{(2 \times 4)+1}{4} + \frac{(5 \times 5)+2}{5} = \frac{9}{4} + \frac{27}{5}$$

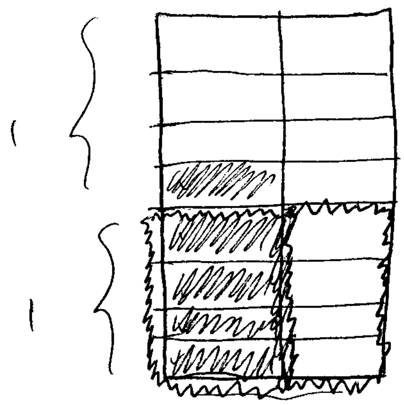
$$\frac{9 \times 5}{4 \times 5} = \frac{45}{20} \qquad \frac{27}{5} \times \frac{4}{4} = \frac{108}{20}$$

$$\frac{45}{20} + \frac{108}{20} = \frac{153}{20} = 7\frac{13}{20}$$

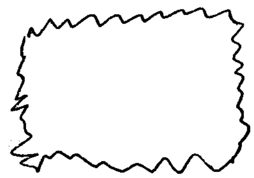
$$\begin{array}{r} 27 \\ \times\ 4 \\ \hline 108 \end{array} \qquad \begin{array}{r} 108 \\ +\ 45 \\ \hline 153 \end{array} \qquad \begin{array}{r} 7 \\ 20\overline{)153} \\ 14\ \\ \hline 13 \end{array}$$

9. **§6.3 Multiplying Fractions**

(a) Using the rectangular area model, compute $\frac{5}{4} \times \frac{1}{2}$. (4 pts)



$\frac{5 \text{ shaded}}{8 \text{ boxes per unit}} = \frac{5}{8}$



indicates the unit.

(b) You need to carpet a room that is 100 inches by 144 inches. The carpet store sells their carpet in square feet. The carpet is \$8 per square foot. How much money do you have to spend on carpet, assuming you buy exactly enough? (There are 12 inches in a foot.) (3pts)

$\frac{100}{\cancel{12}} \times \frac{\overset{12}{\cancel{144}}}{12} = 100 \times 1 = 100$ square feet needed.

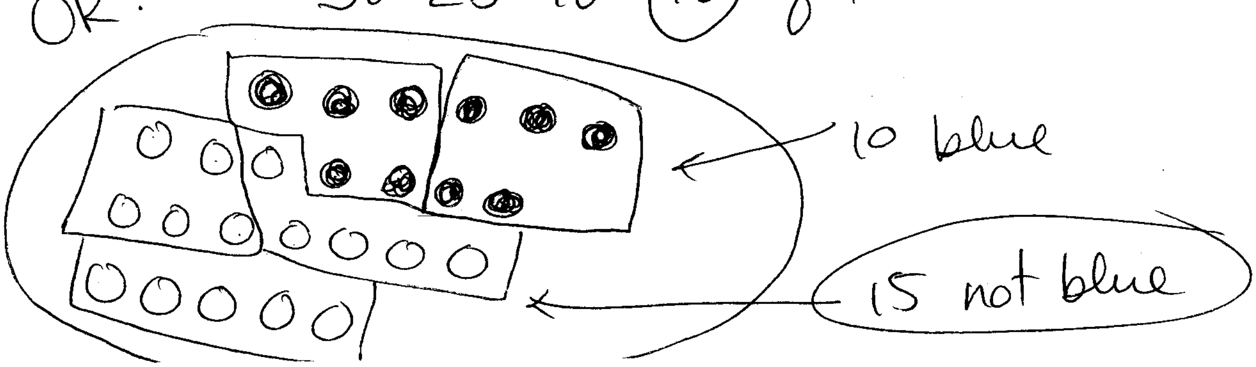
$\$8 \times 100 =$ \$800 for carpet

(c) A bag contains 25 marbles. If $\frac{2}{5}$ of the marbles are blue, how many marbles are **not** blue? (3pts)

$\frac{25}{1} \times \frac{2}{5} = \frac{50}{5} = 10$ of the marbles are blue.

So $25 - 10 = 15$ of the marbles are not blue.

OR:



10. §6.3 **Dividing Fractions**

(a) The following is a proof that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{c}{d}$. Beside each step, justify why ~~each step is~~ valid. (The first line has been done for you.) We start by knowing that if $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$ then we also have $\frac{a}{b} = \frac{e}{f} \times \frac{c}{d}$.(4pts)

Given above $\qquad \frac{a}{b} = \frac{e}{f} \times \frac{c}{d}$

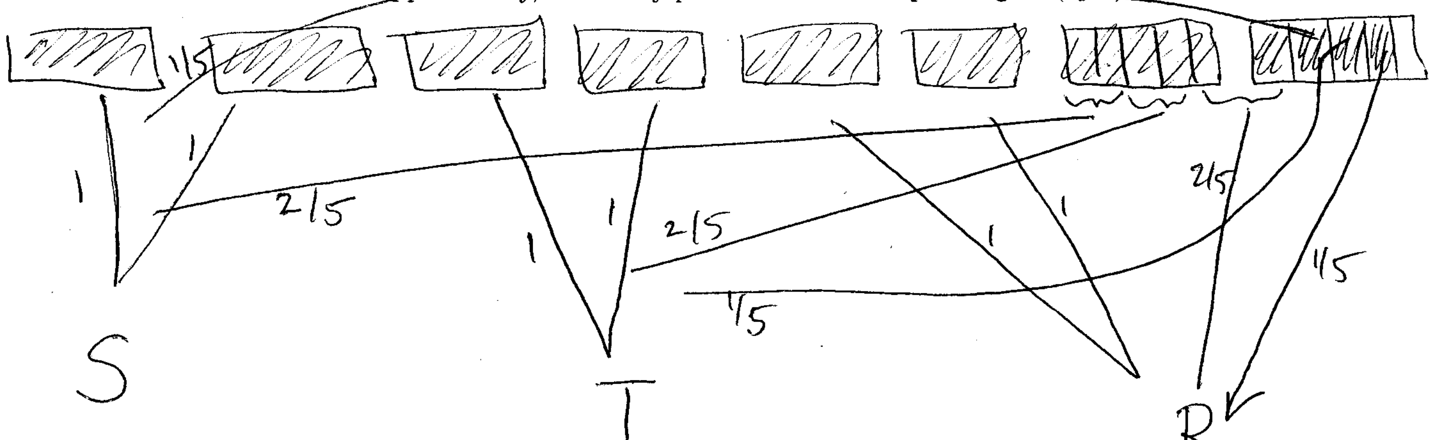
mult. both sides by $\frac{d}{c}$ $\qquad \Rightarrow \frac{a}{b} \times \frac{d}{c} = \left(\frac{e}{f}\frac{c}{d}\right)\frac{d}{c}$

$\frac{c}{d} \times \frac{d}{c} = 1$ $\qquad \Rightarrow \frac{a}{b} \times \frac{d}{c} = \frac{e}{f}$

$\frac{e}{f} = \frac{a}{b} \div \frac{c}{d}$ by $\qquad \Rightarrow \frac{a}{b} \times \frac{d}{c} = \frac{a}{b} \div \frac{c}{d}$

Conclusion $\qquad$ Therefore $\frac{a}{b} \times \frac{d}{c} = \frac{a}{b} \div \frac{c}{d}$.

(b) Draw a diagram to represent and solve the problem:
Suzy, Tiffany, and Rachel want to share $7\frac{4}{5}$ pounds of apples. If they decide to distribute the ~~apples evenly,~~ how many pounds should ~~each person get?~~ (4pts)



each gets $2\frac{3}{5}$

(c) Compute the following and leave your answer in simplest form: (2pts)

(a) $\frac{3}{5} \div \frac{6}{11}$

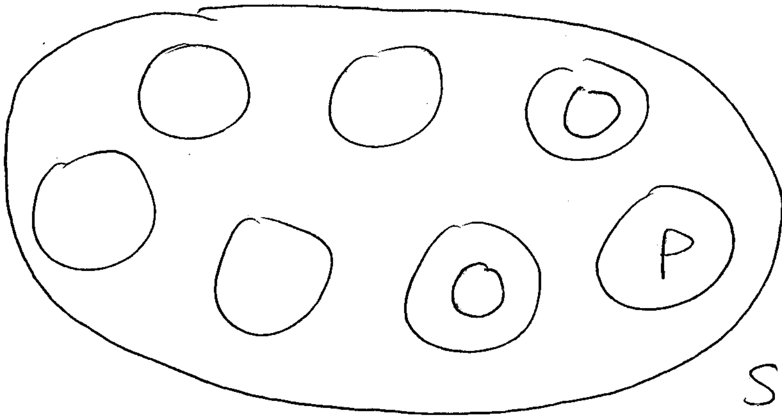
$\frac{\cancel{3}}{5} \times \frac{11}{\cancel{6}_2} = \frac{11}{10}$

(b) $\frac{-4}{5} \div \frac{8}{9}$

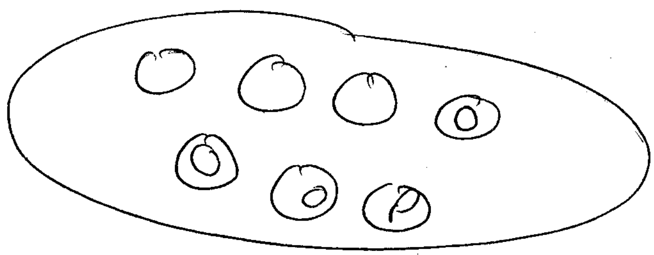
$\frac{-\cancel{4}}{5} \times \frac{9}{\cancel{8}_2} = \frac{-9}{10}$

EXTRA CREDIT

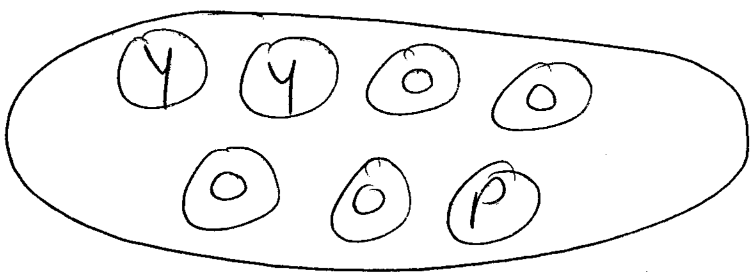
(a) A bag contains yellow, orange and purple marbles. One-seventh of the marbles are purple. There are fifteen more orange marbles than purple marbles. There are half as many yellow marbles as orange marbles. How many marbles are there of each color? (You may use any method you'd like to answer this problem.) (5pts)



Need more orange than purple.
But need half of orange to be yellow
So that $Y+O+P=$ total
So 2 orange circles isn't enough.



3 circles isn't enough.



So, we need 4 orange circles & two Yellow

But how many in each?
We know that Orange marbles are 15 more than purple. So, because there are 3 extra orange circles, 3 orange circles makes up 15 marbles. This means there are 5 marbles per circle. So $P=5$ $O=20$ $Y=10$.