Review for Exam II

Exam II will cover some of the following topics and types of problems.
You should know all of the following number theoretic definitions

- Divisibility

- Factor

- Multiple

- Greatest common divisor

- Least common multiple

- The Division Algorithm

You should know all of the models for division, multiplication and how to use them.

1. You should know the divisibility tests for 2,3,4,5,7,8,10,11, and 13. You should be able to argue that if a and b are divisible by c then $a + b, a - b$, and $a \cdot b$ are all divisible by c.

2. If a and b are whole numbers both divisible by c prove that $a + b$ is also divisible by c. (Hint: First, write down what it means for a and b to be divisible by c)

3. Show that the product of an even and an odd number must be an even number. (Hint: Use the useful representations of even and odd numbers.)

4. Is the number $36,335,936,637$ divisible by 7,13, or 11? Argue with the divisibility tests and show all of your work.

5. Illustrate with a place value diagram the following:

   (a) Subtract 362 from 546.

   (b) Add 642 to 348.

6. Convert 444_{ten} to base five.

7. Convert 321_{five} to base ten.

8. Compute $132_{five} + 221_{five}$ using place value cards and write your answer in base five.

9. Use the scaffold method to perform $5413 \div 12$.

10. (a) Give the definition for the greatest common divisor of the numbers a and b.

    (b) Give the definition for the least common multiple of the numbers a and c.

    (c) Find the greatest common divisor of the numbers 1092 and 525.

    (d) Find the least common multiple of the numbers 1092 and 525.

11. Suppose that $n = a \cdot b$ and that the number c divides n. Is it always true that c divides a or b? Explain your answer.

12. Consider the number abc, abc where the letters a, b, c represent digits. Argue that this number is divisible by 7, 11, and 13.

13. Find the missing whole number.

    (a) $x \div 5 = 7R1$

(b) $47 \div y = 4R3$

14. Mark the following as true or false. Explain each answer with one sentence, giving examples where appropriate.

   (a) Every whole number can be written as a product of prime powers.

   (b) An even number times an even number is always odd.

   (c) One way to find the least common multiple of a and b is to just multiply a and b together.

   (d) If a, m, n are nonzero whole numbers, then $(a^m)^n = a^{m \cdot n}$.

   (e) The set $\{0, 1, 2\}$ is closed under multiplication.

   (f) For any whole number a, $a \div 0 = 0$.

   (g) For any nonzero whole number b, $b/b = 1$.

   (h) The set of prime numbers is finite.

   (i) 0 divides 0.

   (j) $\text{Lcm}(m, n) = m \cdot n / \text{Gcd}(m \cdot n)$.

15. An architect wants to tile a wall that is 12 feet by 16 feet with the largest square tiles possible without having to cut any tiles. What size tiles should he use?

16. Draw an example showing division with remainder zero, and division with a non-zero remainder using number strips.

17. Use the Euclidean Algorithm to determine $\gcd(436, 15)$ and the least common multiple of 436 and 15.

18. Given the number $123,456,78\triangle$ what number could you fill in the triangle to make the number divisible by 10? Is there more than one correct answer?

19. Is 317 prime or composite? (It may be helpful to think of the square root of 317.

20. Is 327 prime or composite? (It may be helpful to think of the square root of 327.

21. You should be able to build the Sieve of Eratosthenes.

22. Should students memorize their multiplication tables? Why or why not? Explain in 2-3 sentences.

23. List all of the factors of 24. You should be able to check that you've listed them all because there is a formula for finding the number of factors of a composite number.

24. How many factors are there of 68? You should be able to use a formula.

Study Tips

1. Begin studying today!

2. Glance through the book/notes and identify any topics that you do not know/understand. Read more about these, and ask me questions.

3. Do the entire review sheet.

4. Do the practice exam.

5. Look over the homework. (All solutions are on the webpage.)

6. Get together with other people and discuss the concepts.

7. Get a good night's sleep the night before and relax the morning of the exam.

Good Luck!